

Mini-symposium on Approximation Algorithms and Complexity in Allocation, Scheduling and Pricing

19 January, room A0.24, Tongersestraat 53, Maastricht

There is no registration fee, but registration is compulsory for organizational purposes. To register, please follow this [link](#).

11:00 Matúš Mihalák (Maastricht University)

From interval scheduling on unrelated machines to black-box approximation of maximization problems over union of independence systems.

11:45 José Correa (Universidad de Chile),

Network pricing: how to induce optimal flows under strategic link operators

12:30 Lunch

13:30 Heiko Röglin (Universität Bonn),

The Alternating Stock Size Problem and the Gasoline Puzzle

14:15 Guido Schäfer (CWI / Vrije Universiteit),

Computing Efficient Nash Equilibria in Congestion Games

16:00 Public Defence by Tim Oosterwijk of his PhD thesis "Approximation Algorithms in Allocation, Scheduling and Pricing" in the aula of Maastricht University, Minderbroedersberg 4-6, Maastricht

Abstracts.

Matúš Mihalák, From interval scheduling on unrelated machines to black-box approximation of maximization problems over union of independence systems.

The interval scheduling problem asks for a maximum number of non-overlapping intervals from a given set of intervals. One can see every interval as a job, and the interval scheduling becomes a single machine scheduling problem, where the interval specifies exactly when the job can be processed on the machine, and we aim to maximize the number of scheduled jobs. In the talk, we will naturally generalize this view to m parallel machines, where every job has exactly one interval on every machine. In general, the m intervals of a job, one per every machine, can be arbitrary, i.e., unrelated. While for $m=1$, the problem can be solved in polynomial time, for $m \geq 2$, the problem is APX-hard. We provide a simple $3/2$ -approximation algorithm for the case of $m=2$. We discuss how the $3/2$ -approximation algorithm can be generalized for any $m > 2$, with the approximation factor approaching 2, while costing multiplicative $m!$ in the runtime (on top of a polynomial in n). We observe and discuss that our approach works for generalizations of the following form: for any maximization problem P (such as our interval scheduling problem on a single machine), consider the composed maximization problem on m copies $P(1), \dots, P(m)$ of P , where we are asked to provide disjoint feasible solutions $S(i)$ to every problem $P(i)$ such that $|S(1)| + |S(2)| + \dots + |S(m)|$ is maximized. This is joint work with Kateřina Böhmová and Enrico Kravina.

José Correa, Network pricing: how to induce optimal flows under strategic link operators

Network pricing games provide a framework for modeling real-world settings with two types of strategic agents: users of the network and owners of the network. Owners of the network post a price for usage of the link they own; users of the network select routes based on price and level of use by other users. The challenge in these games is that there are two levels of competition: one, among the owners to attract users to their link so as to maximize profit; and second, among users of the network to select routes that are cheap yet not too congested. Interestingly, we observe that: (i) an equilibrium may not exist; (ii) it might not be unique; and (iii) the network performance at equilibrium can be arbitrarily inefficient. Our main result is to observe that a slight regulation on the network owners market solves all three issues above. Specifically, if the authority could set appropriate caps (upper bounds) on the tolls (prices) operators can charge then: the game among the link operators has a unique strong Nash equilibrium and the users' game results in a Wardrop equilibrium that achieves the optimal total delay. We call any price vector with these properties a great set of tolls. We then ask, can we compute great tolls that minimize total users' payments? We show that this optimization problem reduces to a linear program in the case of single-commodity series-parallel networks. Starting from the same linear program, we obtain multiplicative approximation results for arbitrary networks with polynomial latencies of bounded degree, while in the single-commodity case we obtain a surprising bound, which only depends on the topology of the network.

This is joint work with Cristóbal Guzmán, Thanasis Lianas, Evdokia Nikolova and Marc Schröder.

Heiko Röglin, The Alternating Stock Size Problem and the Gasoline Puzzle

Given a set S of integers whose sum is zero, consider the problem of finding a permutation of these integers such that:

- (i) all prefix sums of the ordering are non-negative, and
- (ii) the maximum value of a prefix sum is minimized.

Kellerer et al. referred to this problem as the stock size problem and showed that it can be approximated to within $3/2$. They also showed that an approximation ratio of 2 can be achieved via several simple algorithms.

We consider a related problem, which we call the alternating stock size problem, where the numbers of positive and negative integers in the input set S are equal. The problem is the same as above, but we are additionally required to alternate the positive and negative numbers in the output ordering. This problem also has several simple 2-approximations. We show that it can be approximated to within 1.79.

Then we show that this problem is closely related to an optimization version of the gasoline puzzle due to Lovasz, in which we want to minimize the size of the gas tank necessary to go around the track. We present a 2-approximation for this problem, using a natural linear programming relaxation whose feasible solutions are doubly stochastic matrices. Our novel rounding algorithm is based on a transformation that yields another doubly stochastic matrix with special properties, from which we can extract a suitable permutation.

This is joint work with Alantha Newman and Johanna Seif.

Guido Schäfer, Computing Efficient Nash Equilibria in Congestion Games

Congestion games constitute an important class of games which capture many applications in network routing, resource allocation and scheduling. In a seminal paper, Rosenthal (1973) established the existence of pure Nash equilibria in congestion games by exhibiting an exact potential function whose local minima coincide with the set of pure Nash equilibria of the game. This correspondence has helped to shed light on several important aspects of congestion games. In particular, it is exploited crucially to show that finding pure Nash equilibria is a computationally hard problem in general.

We investigate structural properties which allow us to efficiently compute global minima of Rosenthal's potential function. To this aim, we use a polyhedral description to represent the strategy sets of the players and identify two general properties which are sufficient for our result to go through. In addition, we show that the resulting Nash equilibria provide attractive social cost approximation guarantees. Our contributions thus provide an efficient algorithm to find an approximately optimal Nash equilibrium for a large class of polytopal congestion games.

This is joint work with Pieter Kleer.