Price Bubbles

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Overview

1. Some Historical Bubbles

2. A Model of Asset Price Bubbles
   - Basics and Rational Expectations Bubbles
   - A Model of Boundedly Rational Bubbles
   - Long-run Behavior and Numerical Examples
   - Some Remaining Questions

Tulip Bubble

- Tulips introduced to Netherlands in mid 1500's
- Cultivars better adapted to local conditions cultivated later in 1500's
- Some tulips developed stripes of multiple colors, caused by mosaic virus
- Virus passed on through buds
- Virus slows down bud formation
- Some spectacular varieties became famous, but scarce and expensive

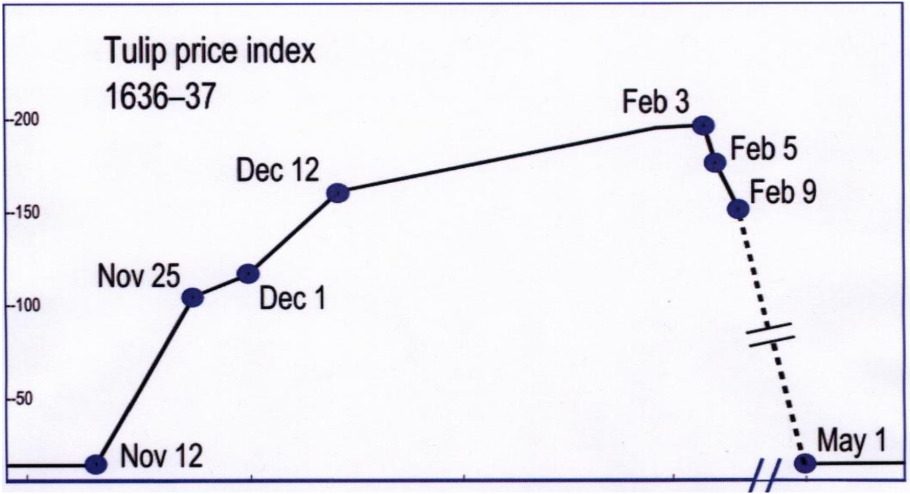
Tulips of the Bubble

Tulips of the Bubble

Tulip Bubble

- Tulip bulbs sold in advance of delivery at end of growing season
- Futures market developed in 1636
- Speculation on futures market drove up futures prices

Tulip Price Index



Tulip Bubble

Was the tulip price behavior really a bubble?

- Very little price data available, most from anti-speculative pamphlets
- Peter Garber argues that it may not have been a bubble, because it is not clear that prices exceeded the "intrinsic value"
- Peter Garber pointed out that prices of other flowers, such as hyacinths, followed similar patterns
- Anne Goldgar points out that, unlike reports in popular literature, relatively small number of people were involved in tulip trade
- Earl Thompson pointed out that buyers on futures market may have known in advance of law passed in 1937 allowing holders of futures to convert them into options with payment of 3.5% penalty
- Other contributing factors: bubonic plague, Thirty Years War

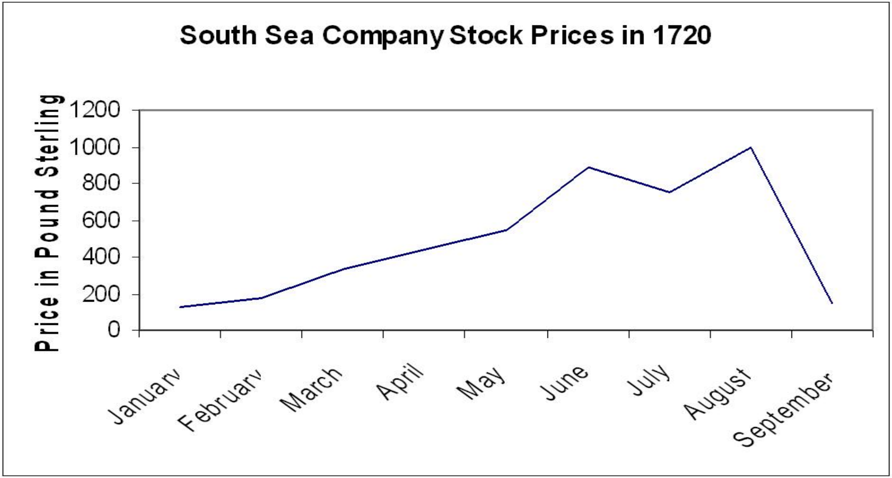
South Sea Bubble

- War of the Spanish Succession
- British Company established in 1711 to trade with Spanish colonies
  - Holders of 10 million pounds of redeemable government war debt exchange it for company stock
  - Government pays lower interest rate to company
  - Government grants company sole right to trade in the South American Spanish colonies (called the South Sea)
- War ended with the Treaty of Utrecht of 1713
- Spanish agreed that British (South Sea Company) may send one trading ship per year, and in addition gives them slave trading contract

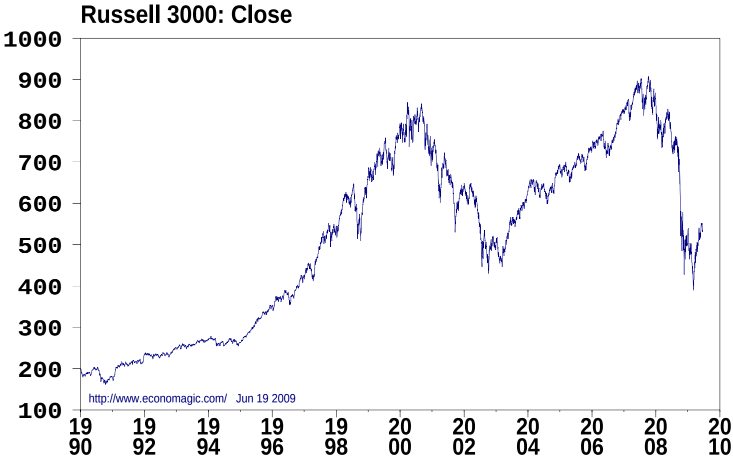
South Sea Bubble

- Another 2 million pounds of government debt added to the agreement in 1717
- Another 16 million pounds of redeemable and irredeemable government debt added to the agreement in 1719
- Company started talking up the company stock price
- Various other scandalous practices
- Another war between Britain and Spain
- High import taxes on slaves in the Spanish colonies make slave trade unprofitable

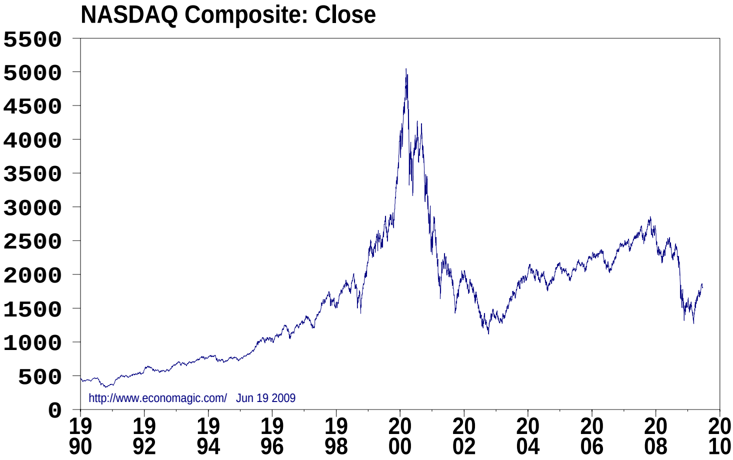
South Sea Bubble



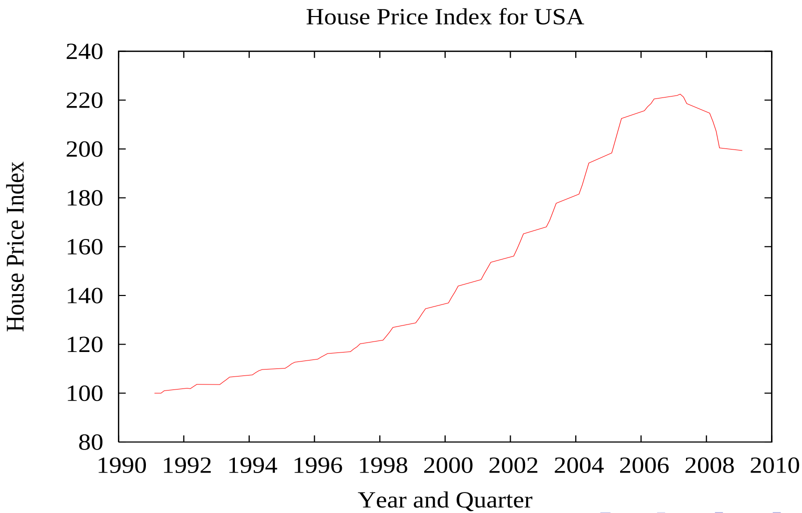
Russell 3000 Stock Index



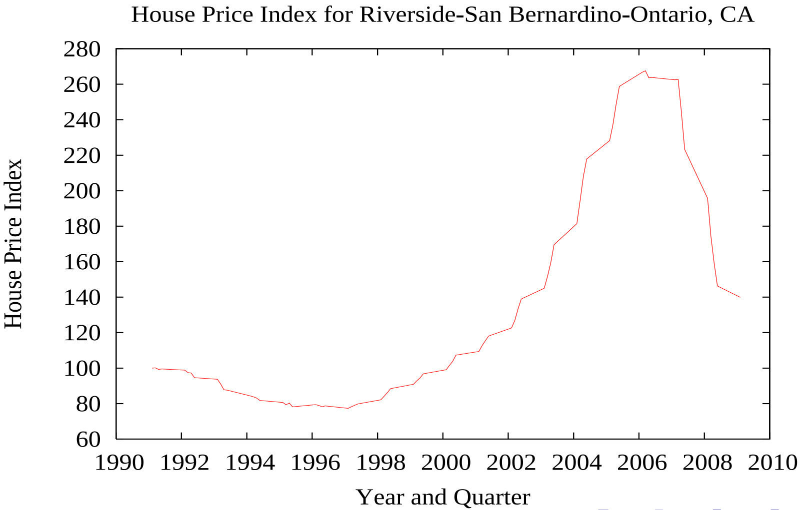
Nasdaq Stock Index



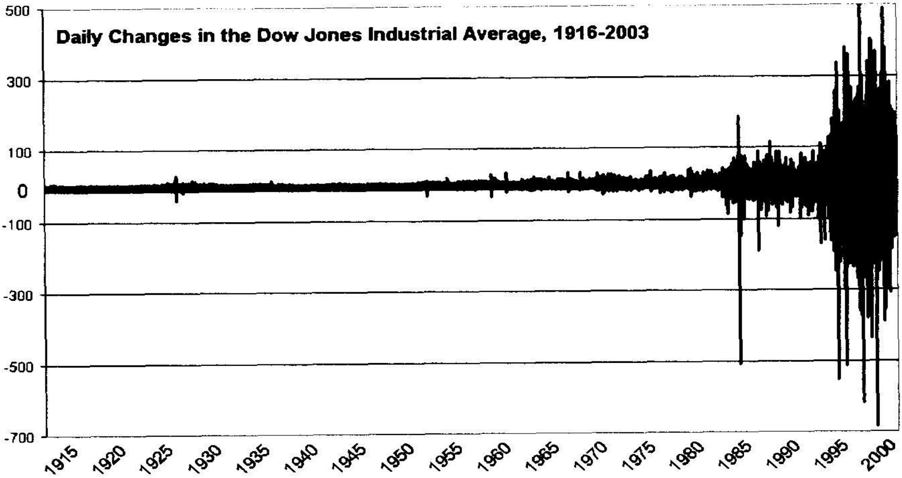
USA House Price Index



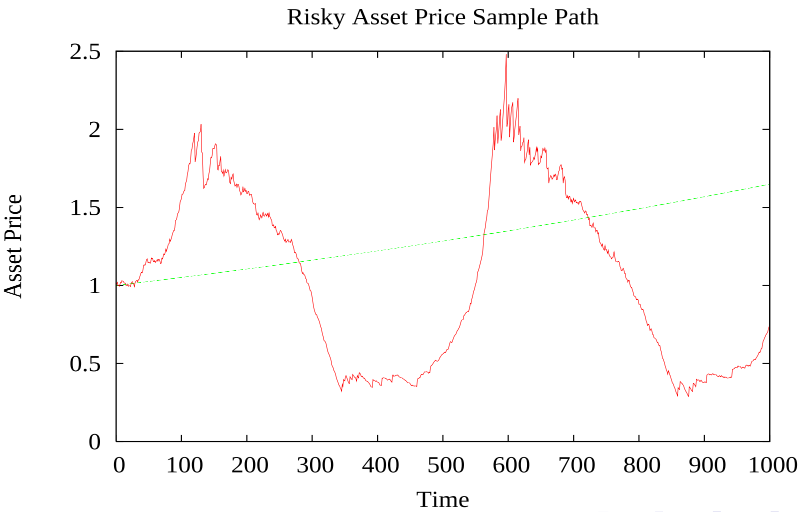
Riverside-San Bernardino-Ontario House Price Index



Increase in Volatility of Dow Jones Index



Sample Path of Risky Asset Prices



Fundamental Value of an Asset

- Discrete time $n = 0, 1, \ldots$
- Asset pays a dividend d_n at time n
- Fundamental value $\bar{p}_n$ at time n (after d_n has been received) of asset:

$$\bar{p}_n \quad := \quad \sum_{i=1}^{\infty} \frac{\mathbb{E}[d_{n+i}|\mathcal{F}_n]}{(1+\tilde{r})^i}$$

where $\tilde{r}$ denotes discount rate used for returns from asset

Market Price and Bubbles under Rational Expectations

- No-arbitrage price p_n at time n (after d_n has been received) satisfies following market clearing equation:

$$p_n(1+\tilde{r}) \quad = \quad \mathbb{E}[p_{n+1} + d_{n+1} | \mathcal{F}_n]$$

- Fundamental value $\bar{p}_n$ satisfies market clearing equation
- However, $p_n = \bar{p}_n + b_n$, where $\mathbb{E}[b_{n+1}|\mathcal{F}_n] = (1+\tilde{r})b_n$, also satisfies the market clearing equation. Such a sequence b_n is sometimes called a rational bubble.

Market Price and Bubbles under Rational Expectations

Shortcomings of above model of rational bubble

- Model does not allow b_n to be negative, that is, price p_n is not allowed to fall below fundamental value $\bar{p}_n$
- Model requires $\mathbb{E}[b_{n+1}|\mathcal{F}_n] = (1 + \tilde{r})b_n$ for all n, that is, (in expectation) bubble grows indefinitely
- Model does not give satisfactory explanation of process by which behavior of decision makers with limited available knowledge, data, and computational power, leads to formation and sustaining of such bubbles
- Empirical support for such models is not strong

A Model of an Asset Price Bubble

Model input

- Risky asset pays a dividend d_n at time n
- Dividend is expected to grow at a rate of r_d per time period

$$\mathbb{E}[d_n] \quad = \quad d_0(1 + r_d)^n$$

- Then fundamental value grows at a rate of r_d per time period

$$\bar{p}_n \quad = \quad \bar{p}_0(1 + r_d)^n$$

as long as $\tilde{r} > r_d$

A Model of an Asset Price Bubble

Price determination

- In each time period, investors can rebalance their portfolios without transaction cost
- Investors base their decision in period n on the price data up to period $n-1$
- Investor's forecast of the price in period $n+1$ is $\hat{p}_{n+1}$
- Market clearing equation in period n:

$$p_n = \frac{1}{1+\tilde{r}}\left[\hat{p}_{n+1} + d_0(1+r_d)^{n+1}\right]$$

Price forecasting

- Suppose investors use exponential smoothing to forecast the growth rate y_n in the price in period n:

$$\hat{y}_n = (1-\alpha)\hat{y}_{n-1} + \alpha \frac{p_{n-1}}{p_{n-2}}$$

  where $\hat{y}_2 = \frac{p_1}{p_0}$
- The corresponding forecast of the price in period $n+1$ is

$$\hat{p}_{n+1} = p_{n-1}\hat{y}_n^2$$

A Model of an Asset Price Bubble

Scaled price

- Recall: fundamental value grows at a rate of r_d per time period

$$\bar{p}_n = \bar{p}_0(1 + r_d)^n$$

- Consider scaled prices

$$\begin{aligned} \pi_n &:= \frac{p_n}{\bar{p}_n} = \frac{p_n}{\bar{p}_0(1 + r_d)^n} \\ \hat{\pi}_n &:= \frac{\hat{p}_n}{\bar{p}_0(1 + r_d)^n} \end{aligned}$$

- In a stable system, π_n should converge to (approximately) 1
- If π_n grows above 1, a positive price bubble forms, and if π_n decreases below 1, a negative price bubble forms

Dynamical system

- In terms of scaled prices

$$\begin{aligned} \hat{y}_n &= (1-\alpha)\hat{y}_{n-1} + \alpha \frac{(1+r_d)\pi_{n-1}}{\pi_{n-2}} \\ \hat{\pi}_{n+1} &= \pi_{n-1} \frac{\hat{y}_n^2}{(1+r_d)^2} \\ \pi_n &= \frac{1+r_d}{1+\tilde{r}} \left[\pi_{n-1} \frac{\hat{y}_n^2}{(1+r_d)^2} + \frac{d_0}{\bar{p}_0} \right] \end{aligned}$$

- If π_n converges, say to $\bar{\pi} \neq 0$, then $\hat{y}_n \to 1 + r_d$
- If $\hat{y}_1$ and p_1/p_0 are large, then $\pi_n \to \infty$ (positive bubble)
- If $\hat{y}_1$ and p_1/p_0 are small, then $\pi_n \to 0$ (negative bubble)

A Model of an Asset Price Bubble

Toward cyclical behavior (bursting the bubble)

- Suppose the price forecast is a weighted combination of the exponential smoothing forecast and the forecast that prices will return to the fundamental value:

$$
\begin{aligned}
\hat{y}_n &= (1-\alpha)\hat{y}_{n-1} + \alpha \frac{p_{n-1}}{p_{n-2}} \\
\hat{p}_{n+1} &= \frac{p_{n-1}\hat{y}_n^2 + \rho\left(\frac{p_{n-1}\hat{y}_n^2}{\bar{p}_0(1+r_d)^{n+1}} - 1\right)^2 \bar{p}_0(1+r_d)^{n+1}}{1 + \rho\left(\frac{p_{n-1}\hat{y}_n^2}{\bar{p}_0(1+r_d)^{n+1}} - 1\right)^2}
\end{aligned}
$$

Dynamical system

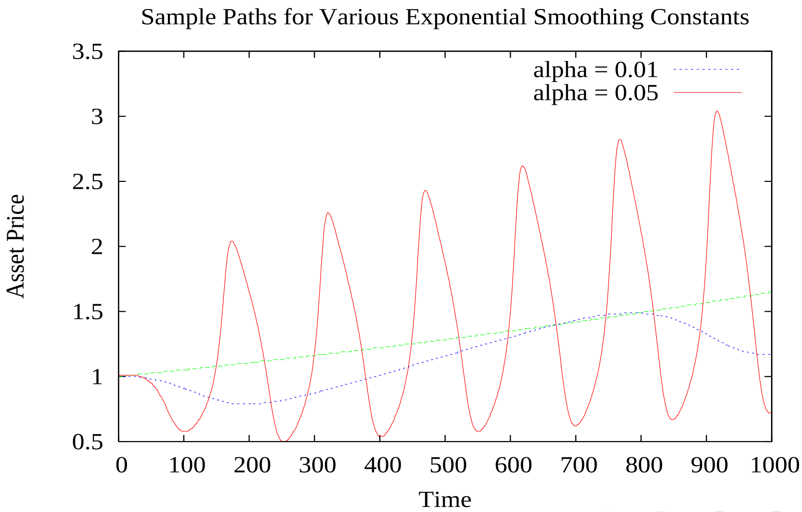
- In terms of scaled prices

$$\hat{y}_n = (1-\alpha)\hat{y}_{n-1} + \alpha\frac{(1+r_d)\pi_{n-1}}{\pi_{n-2}}$$

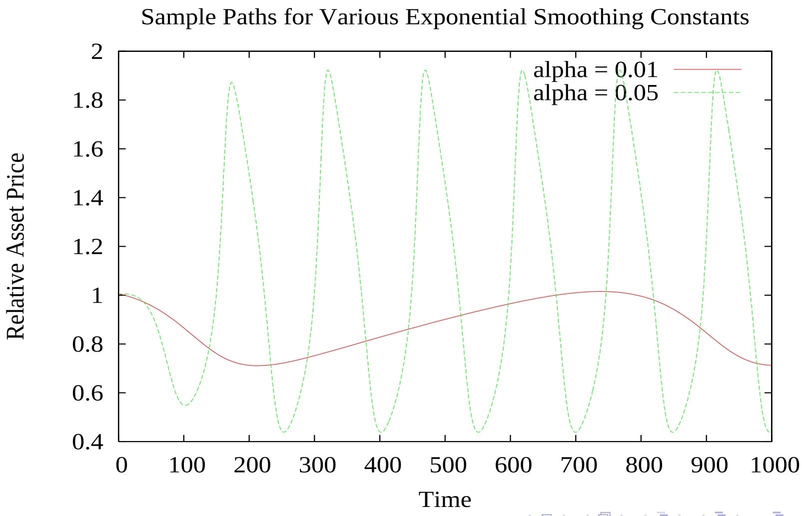
$$\hat{\pi}_{n+1} = \frac{\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 + \rho\left(\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 - 1\right)^2}{1+\rho\left(\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 - 1\right)^2}$$

$$\pi_n = \frac{1+r_d}{1+\tilde{r}}\left[\frac{\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 + \rho\left(\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 - 1\right)^2}{1+\rho\left(\pi_{n-1}\left(\frac{\hat{y}_n}{1+r_d}\right)^2 - 1\right)^2} + \frac{d_0}{\bar{p}_0}\right]$$

Sample Path of Risky Asset Prices



Sample Path of Scaled Risky Asset Prices



Dynamical system

- State variable $x_n = (\pi_n, \pi_{n-1}, \hat{y}_n)$
- Dynamical system $x_{n+1} = f(x_n)$
- If π_n converges, say to $\bar{\pi} \neq 0$, then $\hat{y}_n \to 1 + r_d$, and thus

$$\bar{\pi} = \frac{1 + r_d}{1 + \tilde{r}} \left[\frac{\bar{\pi} + \rho\left(\bar{\pi} - 1\right)^2}{1 + \rho\left(\bar{\pi} - 1\right)^2} + \frac{d_0}{\bar{p}_0} \right]$$

- Solution

$$\bar{\pi} = 1$$

A Model of an Asset Price Bubble

Dynamical system

- Fixed point $\bar{x} = (1, 1, 1 + r_d)$
- The Jacobian $\nabla f(\bar{x})$ at the fixed point $\bar{x}$

$$\nabla f(\bar{x}) = \begin{bmatrix} \frac{2\alpha+1}{\delta+1} & -\frac{2\alpha}{\delta+1} & \frac{2-2\alpha}{r_d\delta+\delta+r_d+1} \\ 1 & 0 & 0 \\ \alpha\,(r_d+1) & -\alpha\,(r_d+1) & 1-\alpha \end{bmatrix}$$

where $\delta := d_0/\bar{p}_0$

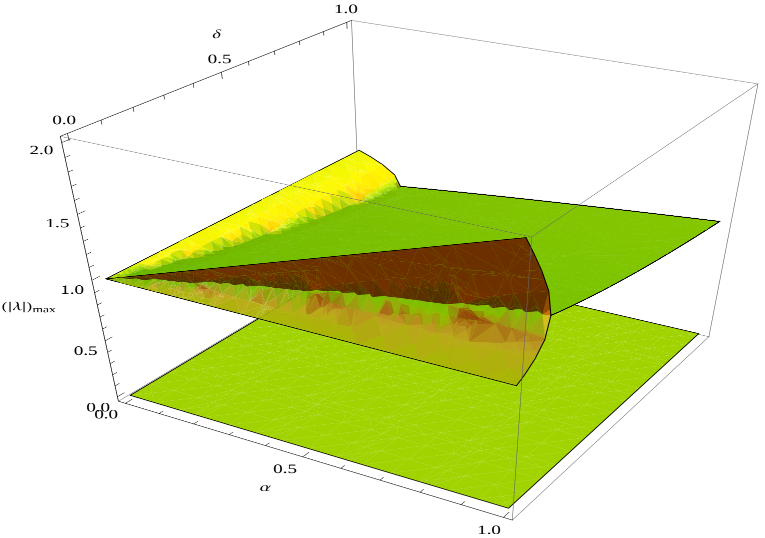
- The eigenvalues of $\nabla f(\bar{x})$ are

$$v_1 = 0$$

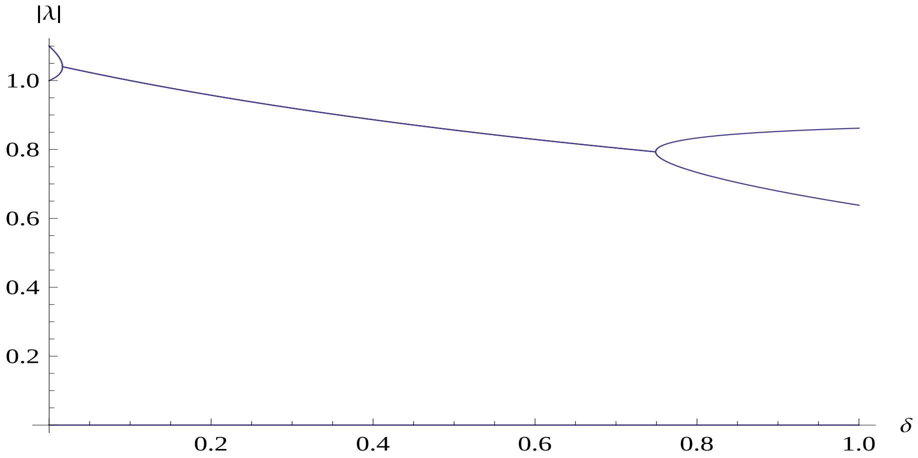
$$v_2 = \frac{\alpha(-\delta) - \sqrt{(\alpha\delta - \alpha - \delta - 2)^2 - 4(\alpha+1)(\delta+1)} + \alpha + \delta + 2}{2(\delta+1)}$$

$$v_3 = \frac{\alpha(-\delta) + \sqrt{(\alpha\delta - \alpha - \delta - 2)^2 - 4(\alpha+1)(\delta+1)} + \alpha + \delta + 2}{2(\delta+1)}$$

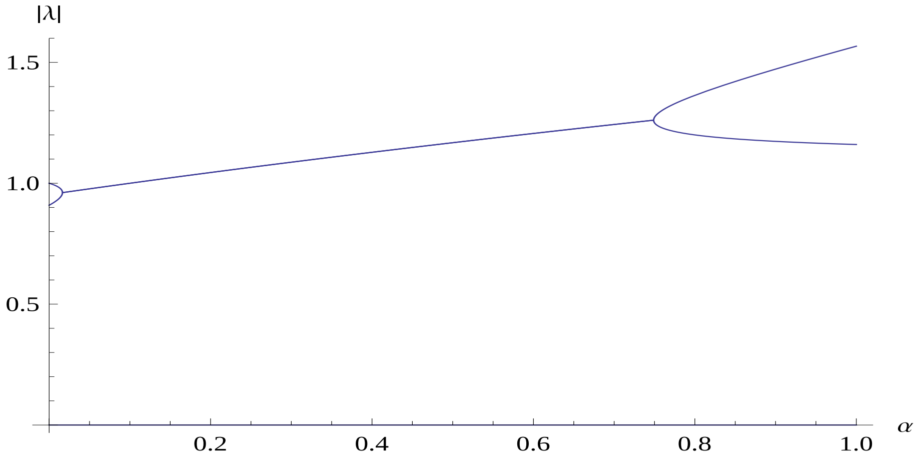
Eigenvalues at Fixed Point



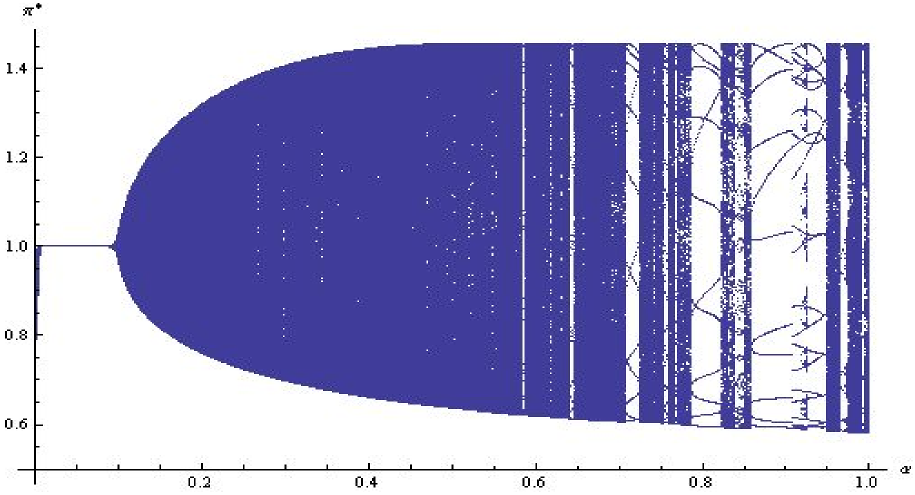
Eigenvalues at Fixed Point



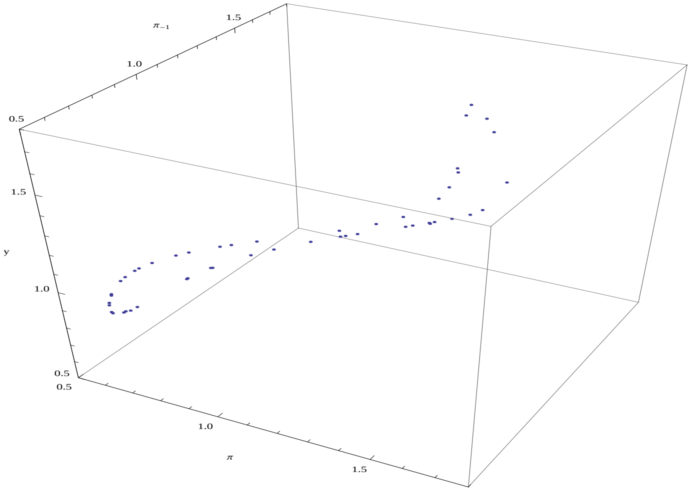
Eigenvalues at Fixed Point



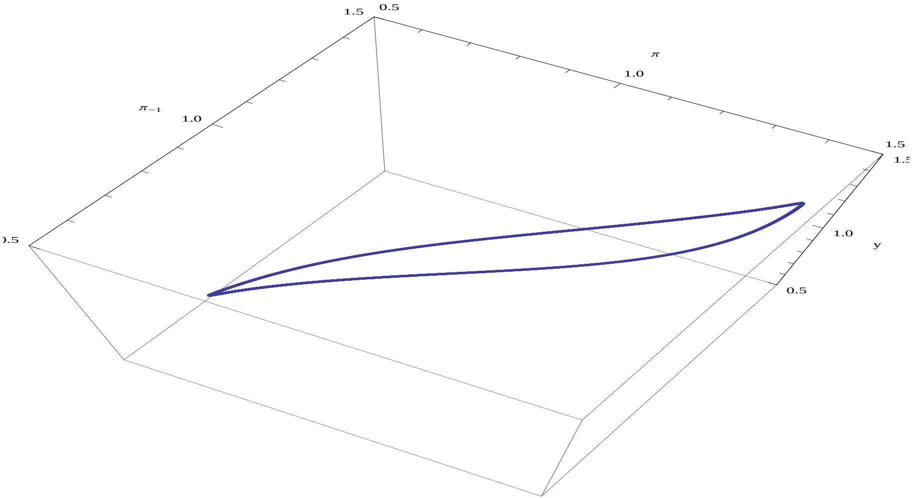
Bifurcation Diagram



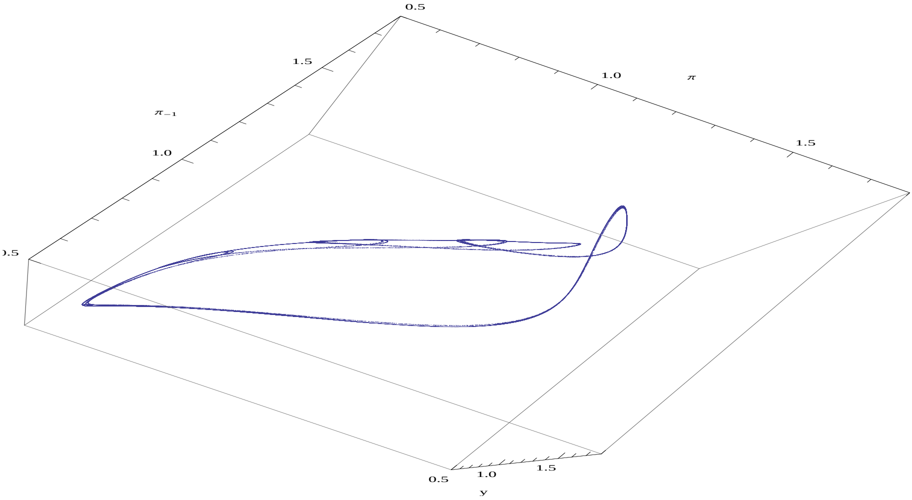
Limit Set of Dynamical System



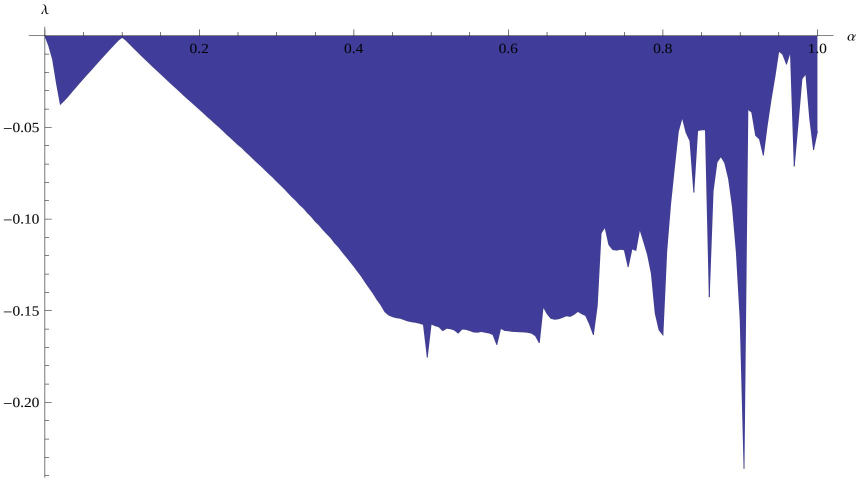
Limit Set of Dynamical System



Limit Set of Dynamical System



Lyapunov Exponents



Some Remaining Questions

- Prove convergence to a limit set
- Establish whether the limit set is uncountable for some parameter values
- Can the limit set be something other than a finite set or a 1-dimensional manifold?