Personnel planning for care-at-home service facilities

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Joint work with Paulien Out & Maarten Meersbergen

Care-at-home services

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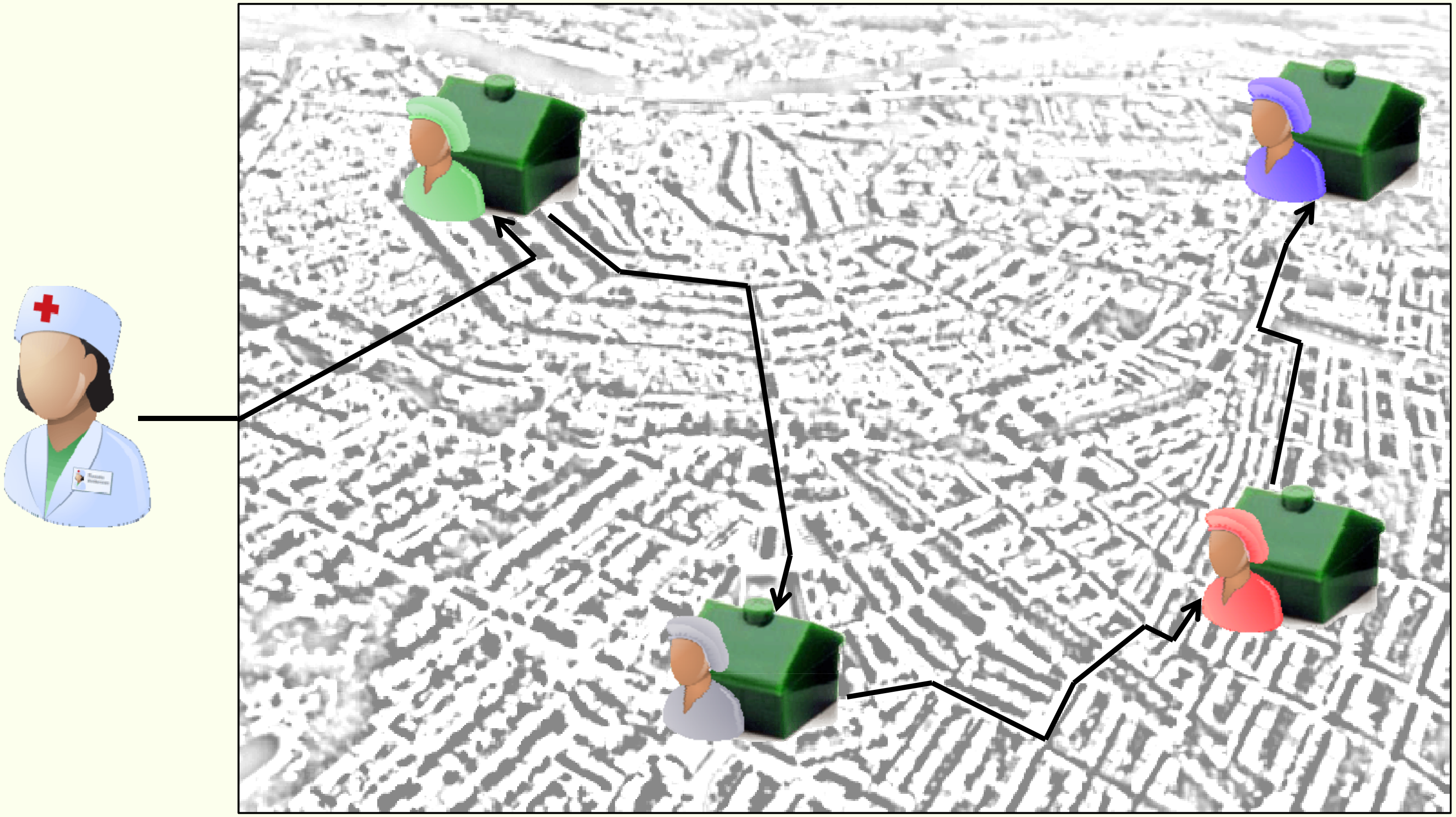
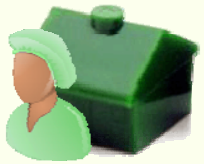
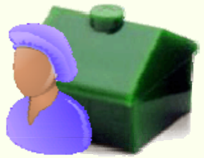
Care-at-home services



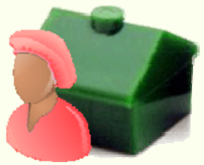
Care-at-home services



needs care for 2 hours a week for 5 consecutive weeks



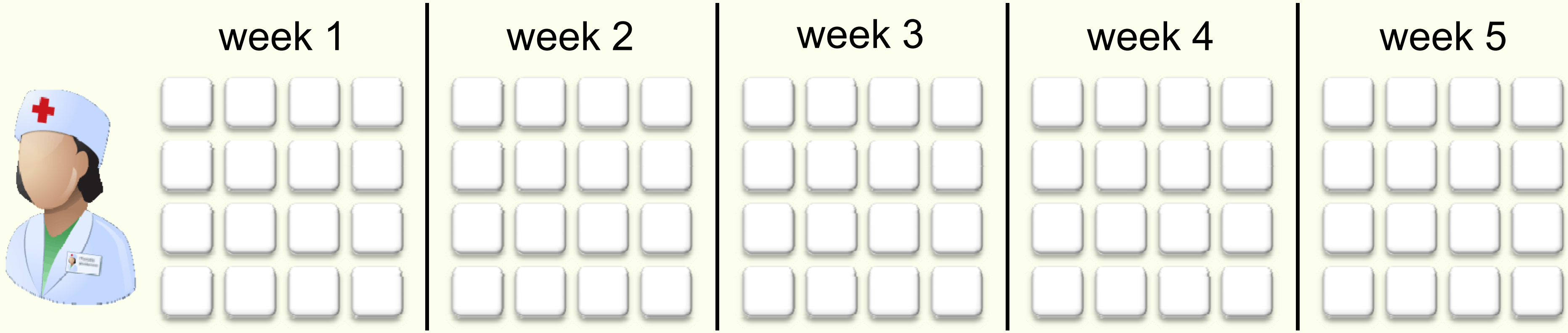
needs care for 3.5 hours a week for 3 consecutive weeks



needs care for 4 hours a week for 4 consecutive weeks

needs care for 2.5 hours a week for 5 consecutive weeks

Care-at-home services



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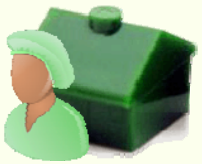

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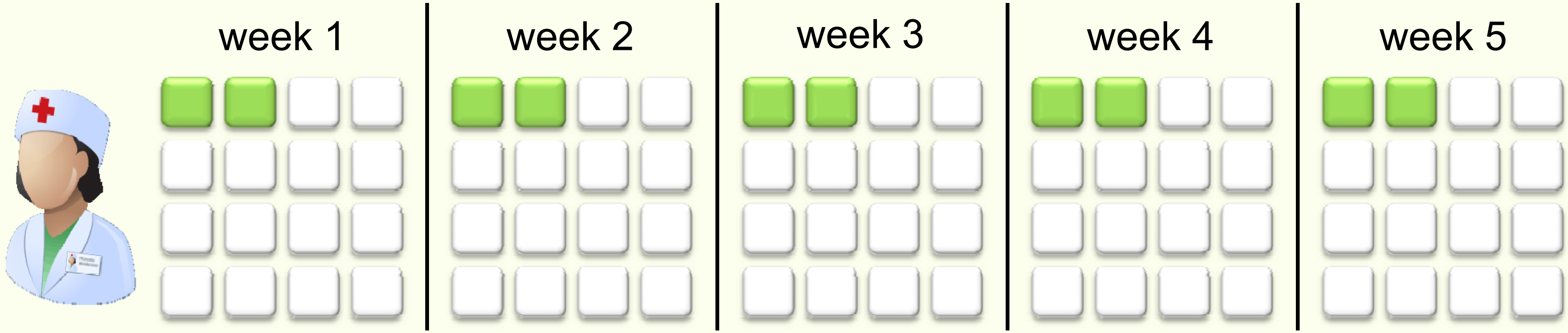

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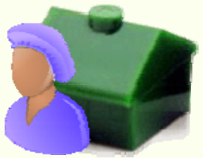

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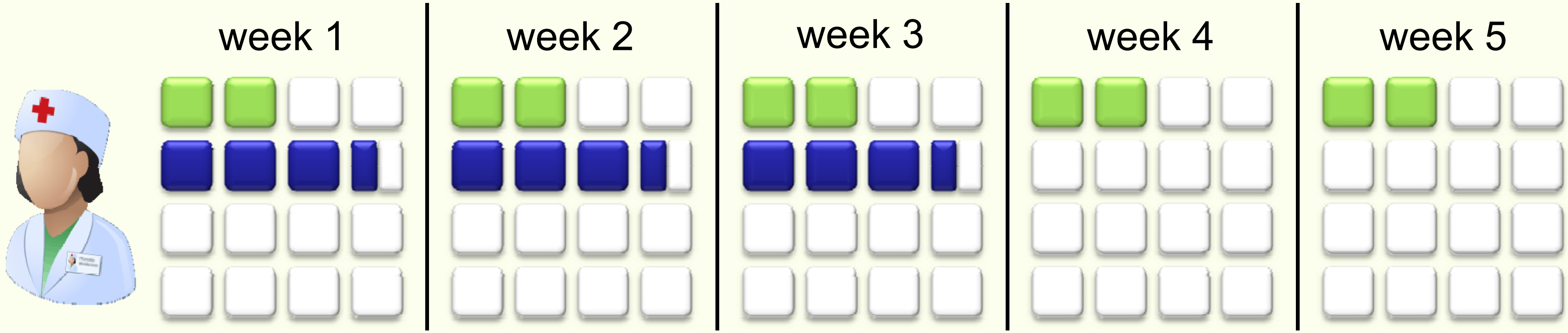

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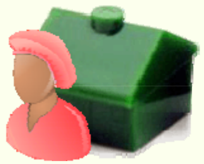

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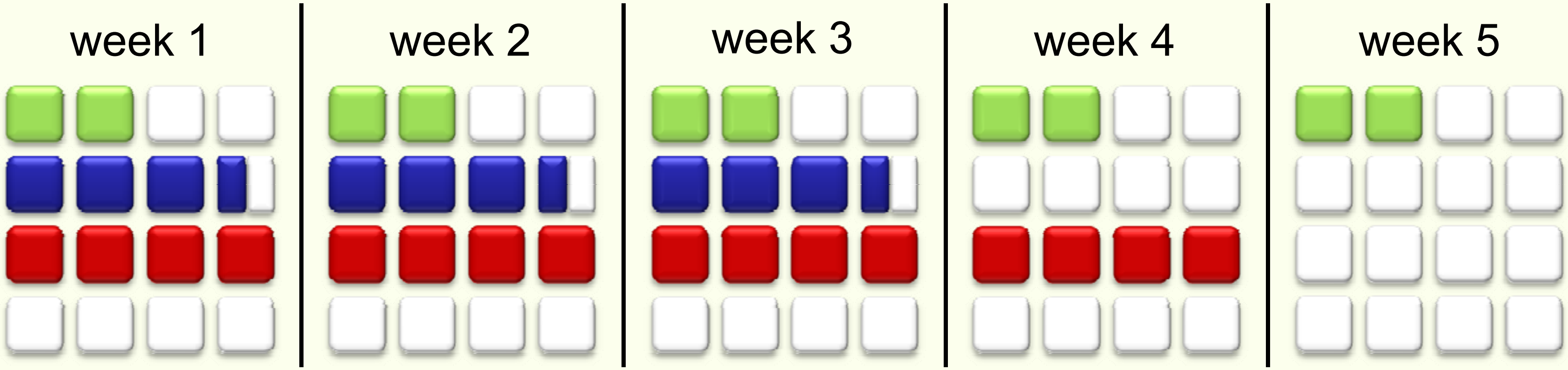

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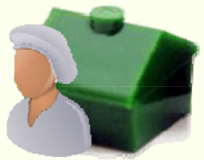
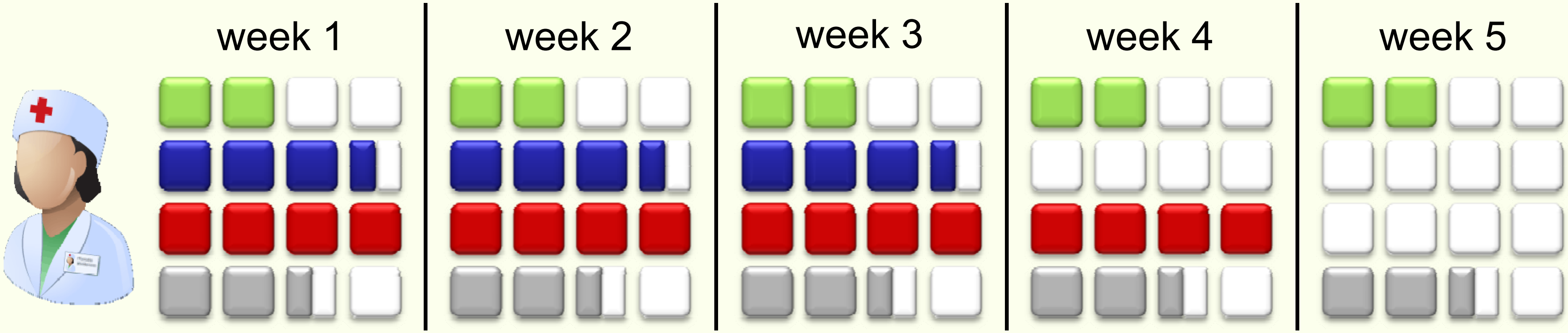


needs care for 2.5 hours a week for 5 consecutive weeks



The care-at-home planning problem

Patient scheduling problem

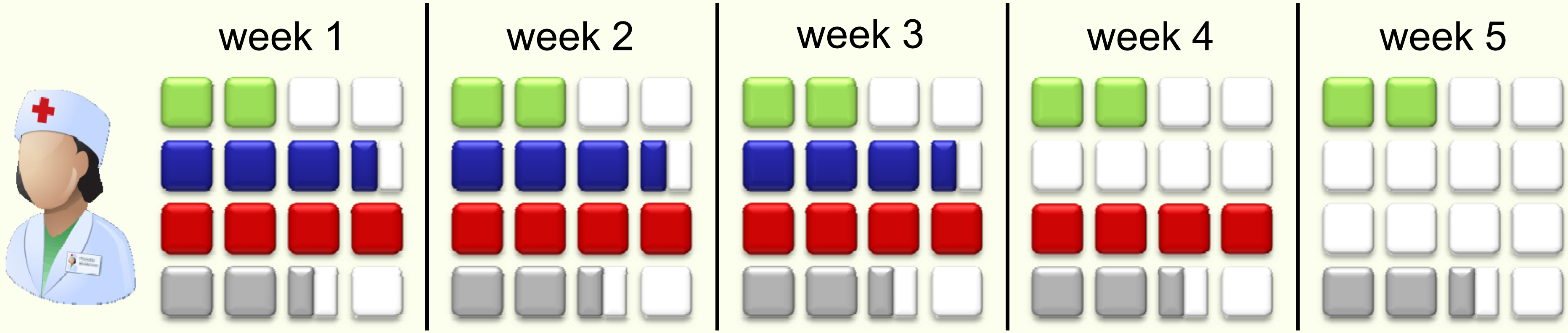
- Do we accept all patients?
- Do we allow for waiting if no capacity is available?

Personnel planning problem

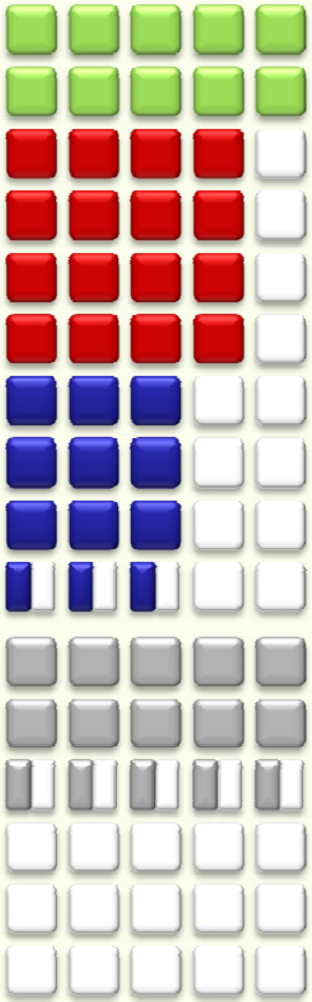
- What size of the workforce achieves
  - acceptable blocking percentages
  - reasonable waiting times
- The solution depends heavily on the patient scheduling problem

The care-at-home planning problem

objective: create a tractable model for care-at-home services



The care-at-home planning problem



arriving patient claims 4 servers for 5 periods

arriving patient claims 8 servers for 4 periods

arriving patient claims 7 servers for 3 periods

arriving patient claims 5 servers for 5 periods

transformation: 0.5 hours of capacity denotes 1 server

The care-at-home model

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- Patients arrive according a Poisson process with rate λ
- There are S servers (capacity units) available per week
- There are K types of patients
- An arriving patient is of type k with probability p_k
- A patient of type k demands c_k care-hours per week
- The number of consecutive care-weeks follows an exponential distribution with parameter μ_k
- Rejecting a customer of type k brings forth costs R_k

The care-at-home model

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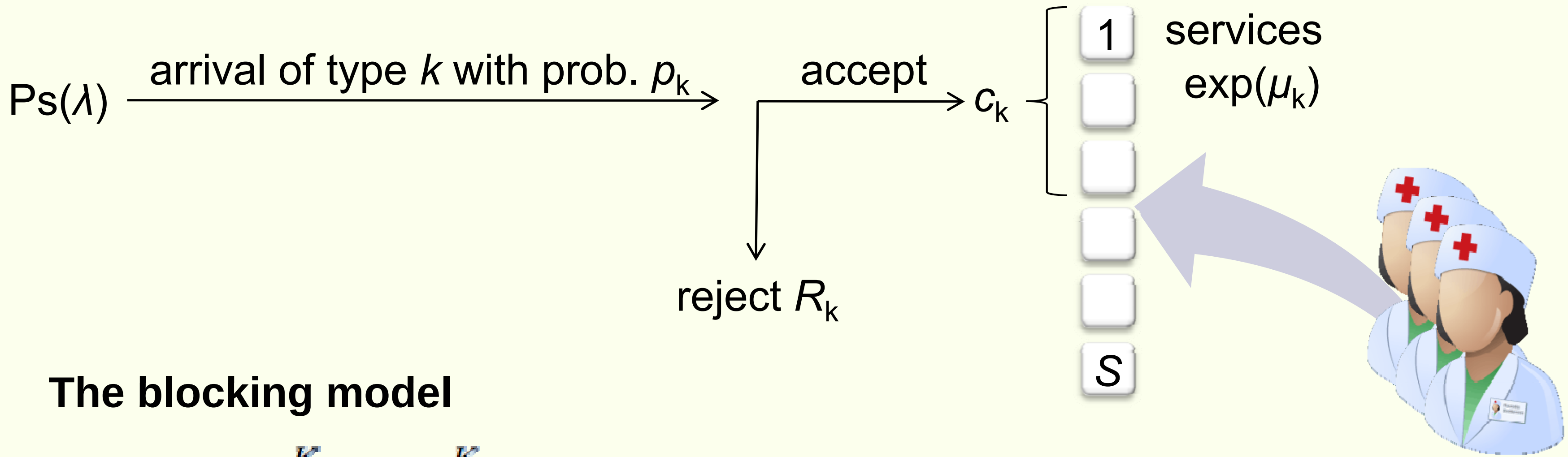

The blocking model

$$g + V(\vec{x}) = \sum_{k=1}^{K} x_k + \sum_{k=1}^{K} \lambda p_k \Big[\mathbb{1}_{\{\sum_{l=1}^{K} c_l \cdot x_l > S - c_k\}} [V(\vec{x}) + R_k] +$$

$$\mathbb{1}_{\{\sum_{l=1}^{K} c_l \cdot x_l \leq S - c_k\}} \min\{V(\vec{x}) + R_k, V(\vec{x} + e_k)\} \Big] + \sum_{k=1}^{K} x_k \mu_k V(\vec{x} - e_k) +$$

$$\Big(1 - \lambda - \sum_{k=1}^{K} x_k \mu_k\Big) V(\vec{x}).$$

The care-at-home model

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Literature

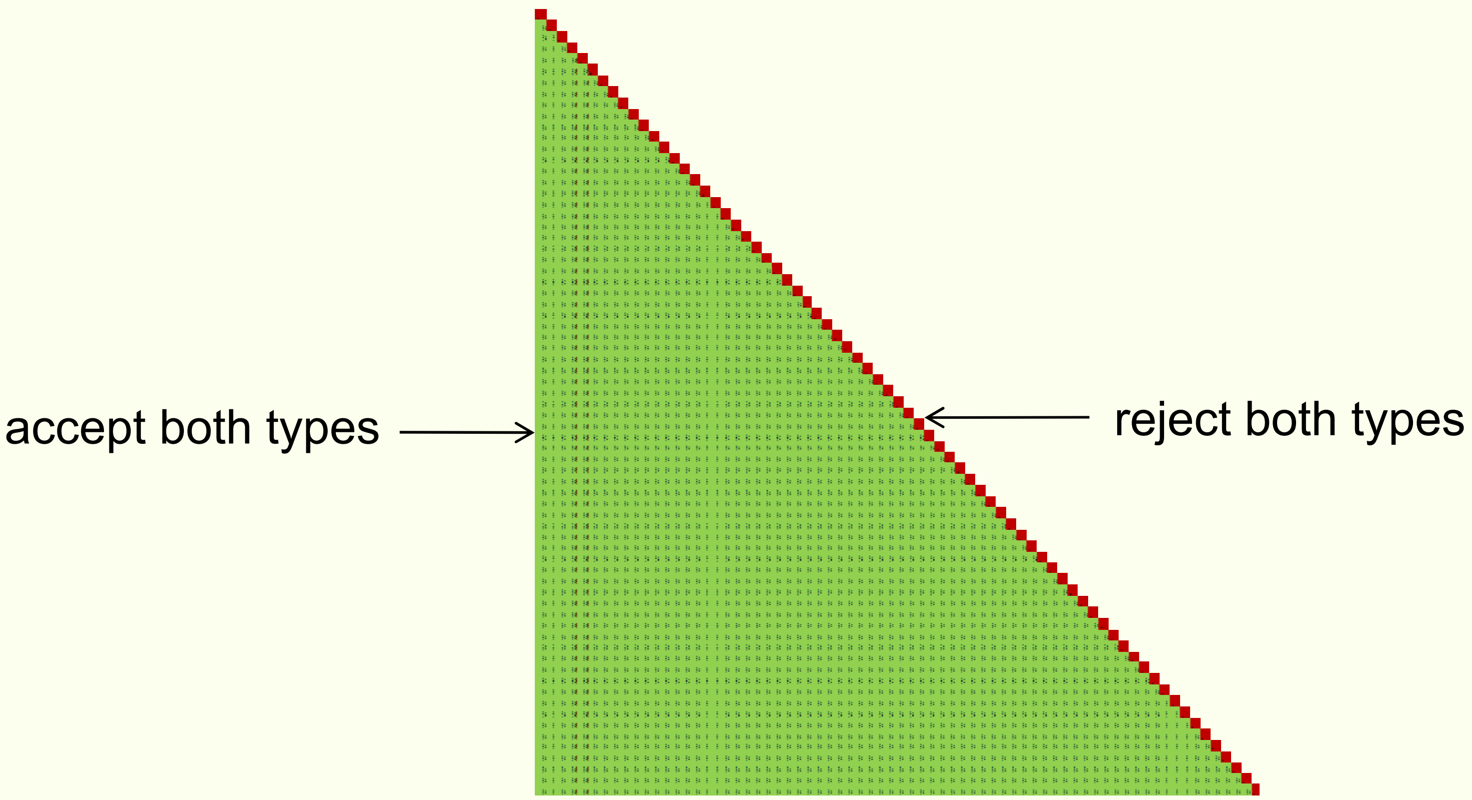
- Stochastic knapsack problem
- Multi-rate blocking model

Results

- $c_i = 1$, $\mu_i = \mu \rightarrow$ Trunk reservation policy (Miller, 1969)
- Trunk reservation not optimal (Ross and Tsang, 1989)
- Monotonicity, fluid approach (Altman et al., 2001)

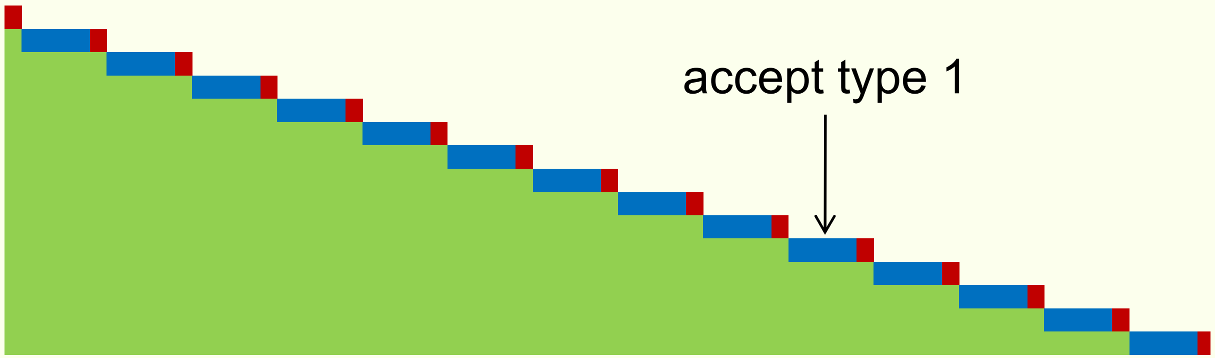
Numerical experiments

$K = 2$, $S = 70$, $\lambda = 1$, $p = (0.5, 0.5)$, $\mu = (1, 1)$, $r = (1, 1)$, and $c = (1, 1)$



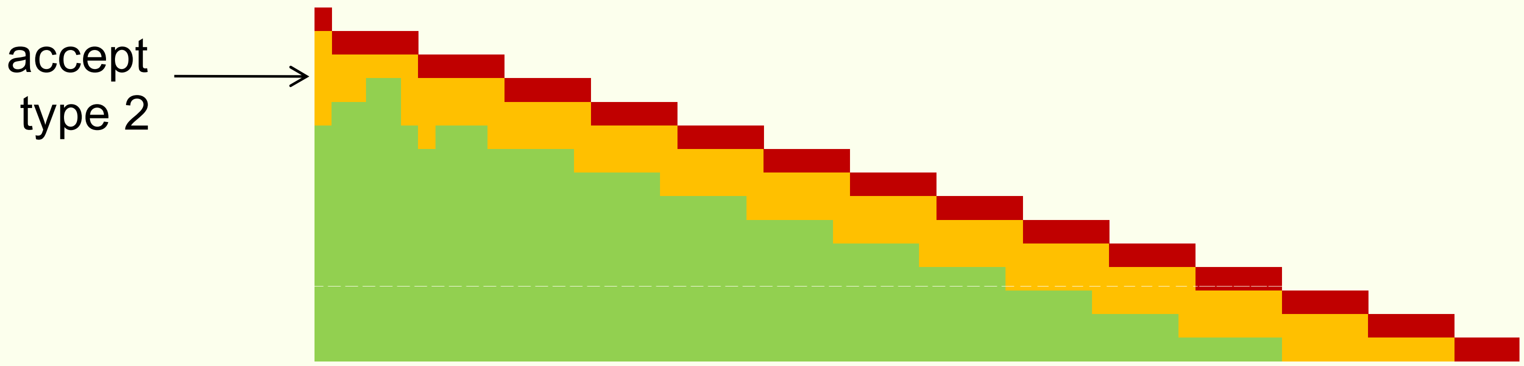
Numerical experiments

$K = 2$, $S = 70$, $\lambda = 1$, $p = (0.5, 0.5)$, $\mu = (1, 1)$, $r = (1, 1)$, and $c = (1, 5)$



Numerical experiments

$K = 2$, $S = 70$, $\lambda = 1$, $p = (0.5, 0.5)$, $\mu = (1, 1)$, $r = (1, 2)$, and $c = (1, 5)$



Numerical experiments

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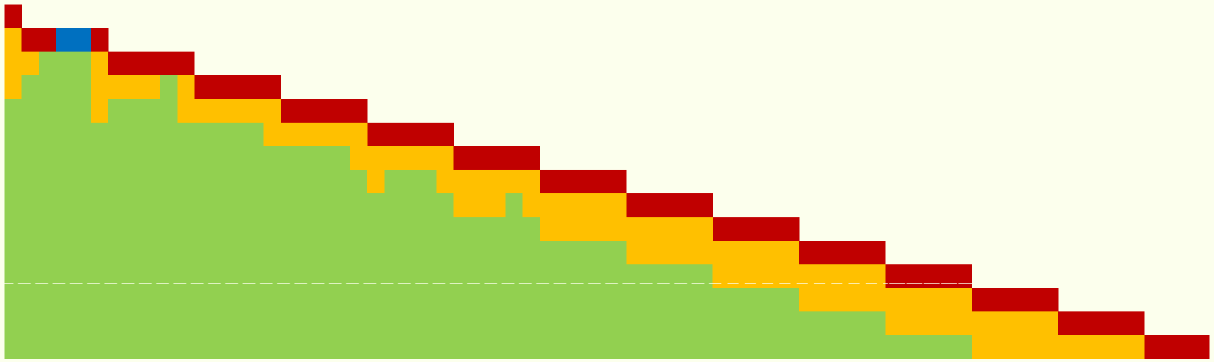
$K = 2$, $S = 70$, $\lambda = 1$, $p = (0.5, 0.5)$, $\mu = (1, 3)$, $r = (1, 2)$, and $c = (1, 5)$



The care-at-home model

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The delay model

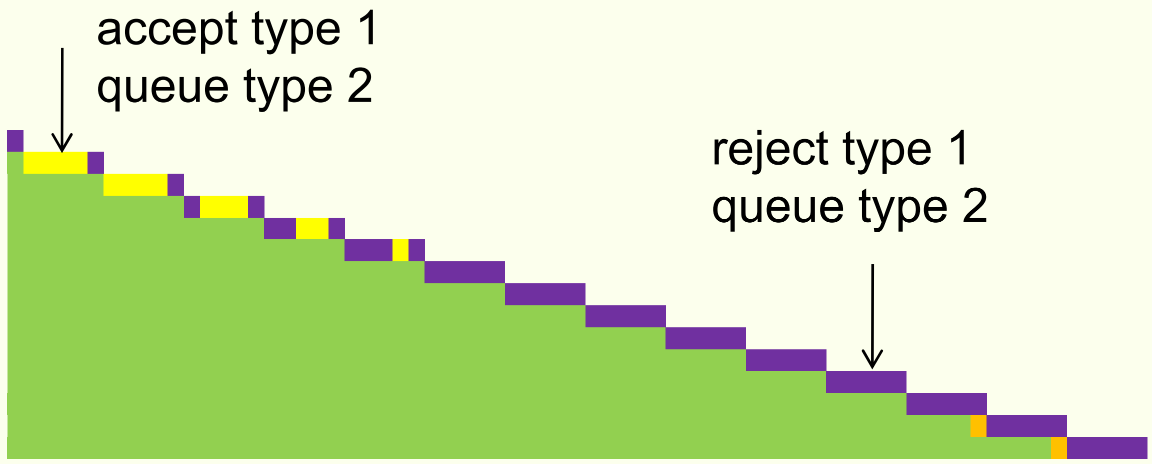
$$g + V(\vec{x},\vec{q}) = \sum_{k=1}^{K}(x_k + q_k) + \sum_{k=1}^{K}\lambda p_k H_a(\vec{x},\vec{q},k) + \sum_{k=1}^{K} x_k \mu_k H_d(\vec{x} - e_k,\vec{q}) + \Big(1 - \lambda - \sum_{k=1}^{K} x_k \mu_k\Big) V(\vec{x},\vec{q}),$$

$$H_a(\vec{x},\vec{q},k) = \begin{cases} V(\vec{x},\vec{q}) + R_k, & \text{if } \sum_{l=1}^{K} c_l \cdot x_l > S - c_k, \sum_{k=1}^{K} q_k = B, \\ \min\{V(\vec{x},\vec{q}) + R_k, V(\vec{x},\vec{q}+e_k)\}, & \text{if } \sum_{l=1}^{K} c_l \cdot x_l > S - c_k, \sum_{k=1}^{K} q_k < B, \\ \min\{V(\vec{x},\vec{q}) + R_k, V(\vec{x}+e_k,\vec{q})\}, & \text{if } \sum_{l=1}^{K} c_l \cdot x_l \le S - c_k, \sum_{k=1}^{K} q_k = B, \\ \min\{V(\vec{x},\vec{q}) + R_k, V(\vec{x},\vec{q}+e_k), V(\vec{x}+e_k,\vec{q})\}, & \text{otherwise}, \end{cases}$$

$$H_d(\vec{x},\vec{q}) = \min\Big\{V(\vec{x},\vec{q}), V(\vec{x}+e_k,\vec{q}-e_k) \,|\, k = 1,\ldots,K, q_k > 0, \sum_{l=1}^{K} c_l \cdot x_l > S - c_k\Big\}.$$

Numerical experiments

$K = 2$, $S = 70$, $B = 20$, $\lambda = 1$, $p = (0.5, 0.5)$, $\mu = (1, 3)$, $r = (1, 2)$, and $c = (1, 5)$



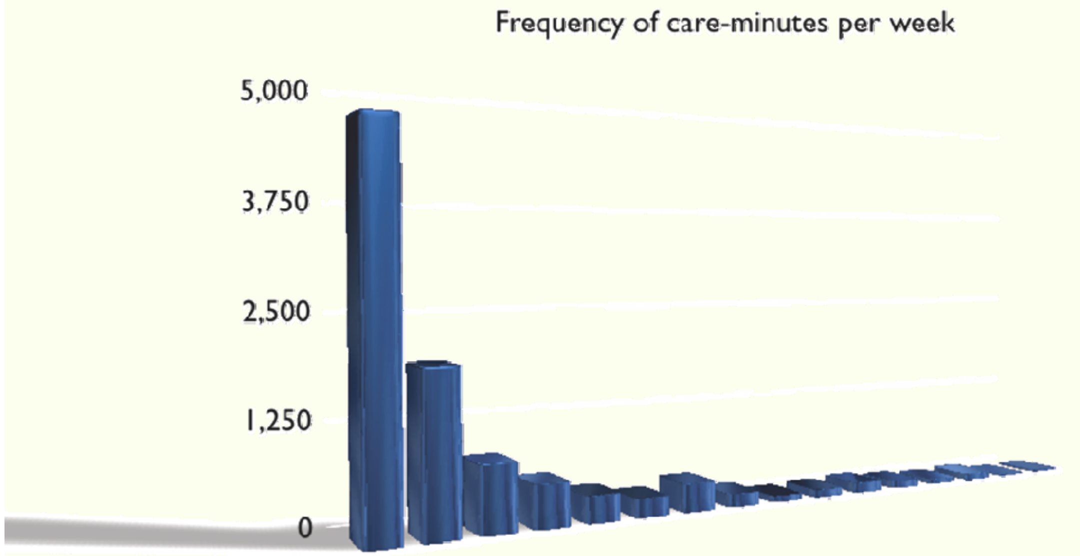
Numerical experiments

- Model with delay is high dimensional
- *n* customer classes lead to 2*n*-dimensional state
- The policy seems to be more structured
- Implemented in MPI-CC on 128-CPU cluster computer

Case study

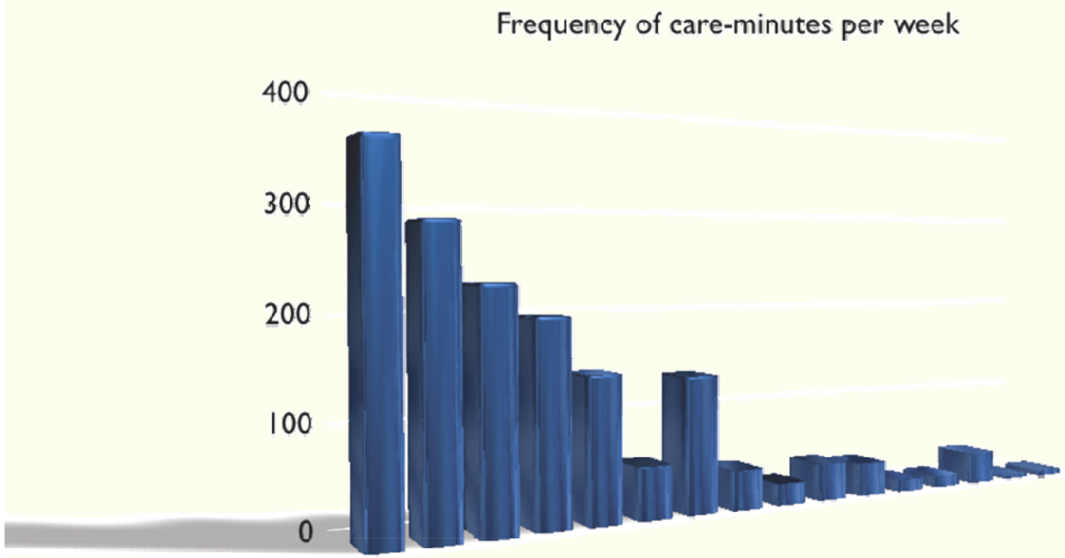
Data from care-at-home facility: personal care (PC)

- λ = 4.9 patients per day
- average number of care-minutes per week: 89.2
- 1 / μ = 24.9 care-weeks



Data from care-at-home facility: medical care (MC)

- λ = 0.83 patients per day
- average number of care-minutes per week: 124
- 1 / μ = 31.8 care-weeks



Case study

Personnel planning: no uncertainty

- PC: 4.9 x 89.2 x 24.9 = 181.4 hours
- MC: 0.83 x 124 x 31.8 = 54.5 hours
- Total net capacity needed: 235.9 hours = 5.9 fte

Personnel planning: with variability

- Total net capacity: 12.7 fte

Future work

- Approximate relative value function using ADP
- Add skill-level of personnel in decision making
- Patients may change their type after acceptance
- Integrate travel times and route planning

Questions?