A Model of Price Bubbles and Business Cycles: Extended Abstract

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December 23, 2009

Introduction Consider a simple model of investors who periodically rebalance their portfolios by buying or selling a risky asset or an investment that pays a fixed rate of return. We consider a discrete time model with time index $n = 0, 1, \ldots$. The risk-free asset provides a guaranteed interest rate of r per time period. The returns from the risky asset consist of both dividends and price changes. The risky asset pays a dividend d_n in period n. Suppose that the return grows at a rate of r_d per time period, that is

$$\mathbb{E}[d_n] \quad = \quad d_0(1 + r_d)^n$$

(for simplicity, we omit conditional expectation here). The fundamental value of the risky asset is equal to the net present value of future returns. Specifically, let $\bar{p}_n$ denote the fundamental value of the risky asset at time n just after the return d_n has been paid out, and let $\tilde{r} > r_d$ denote the discount rate used for the returns from the risky asset. Then

$$\bar{p}_n \quad := \quad \sum_{i=1}^{\infty} \frac{d_{n+i}}{(1+\tilde{r})^i} \quad = \quad \sum_{i=1}^{\infty} d_0 \frac{(1+r_d)^{n+i}}{(1+\tilde{r})^i}$$

It follows that the fundamental value of the risky asset at time n is given by $\bar{p}_n = \bar{p}_0(1 + r_d)^n$.

A well-known result is that, under a number of conditions, the no-arbitrage price p_n at time n satisfies the following market clearing equation:

$$p_n(1 + r) \quad = \quad \mathbb{E}[\hat{p}_{n+1} + d_{n+1}]$$

It can be seen that if $r = \tilde{r}$, then $\bar{p}_n$ satisfies the market clearing equation. However, in that case the sequence $p_n = \bar{p}_n + b_n$, where $\mathbb{E}[b_{n+1}] = (1 + r)b_n$, also satisfies the market clearing equation. Such a sequence b_n is sometimes called a rational bubble. Much research has been devoted to the study of such bubble sequences, to investigate conditions under which such sequences may exist or cannot exist, to characterize such sequences, and to detect such sequences in data. It is acknowledged that these models suffer from various shortcomings. First, the model does not allow b_n to be negative, that is, the price p_n is not allowed to fall below the fundamental value $\bar{p}_n$. Second, the model requires the bubble to grow indefinitely. Such models are sometimes accompanied by verbal acknowledgements that the bubble bursts at some point, but as mentioned, the model requires that $\mathbb{E}[b_{n+1}] = (1 + r)b_n$. Third, the model does not give a satisfactory explanation of the process by which the behavior of decision makers with limited available knowledge, data, and computational power, leads to the formation and sustaining of such bubbles. Fourth, the empirical support for such models is not strong.

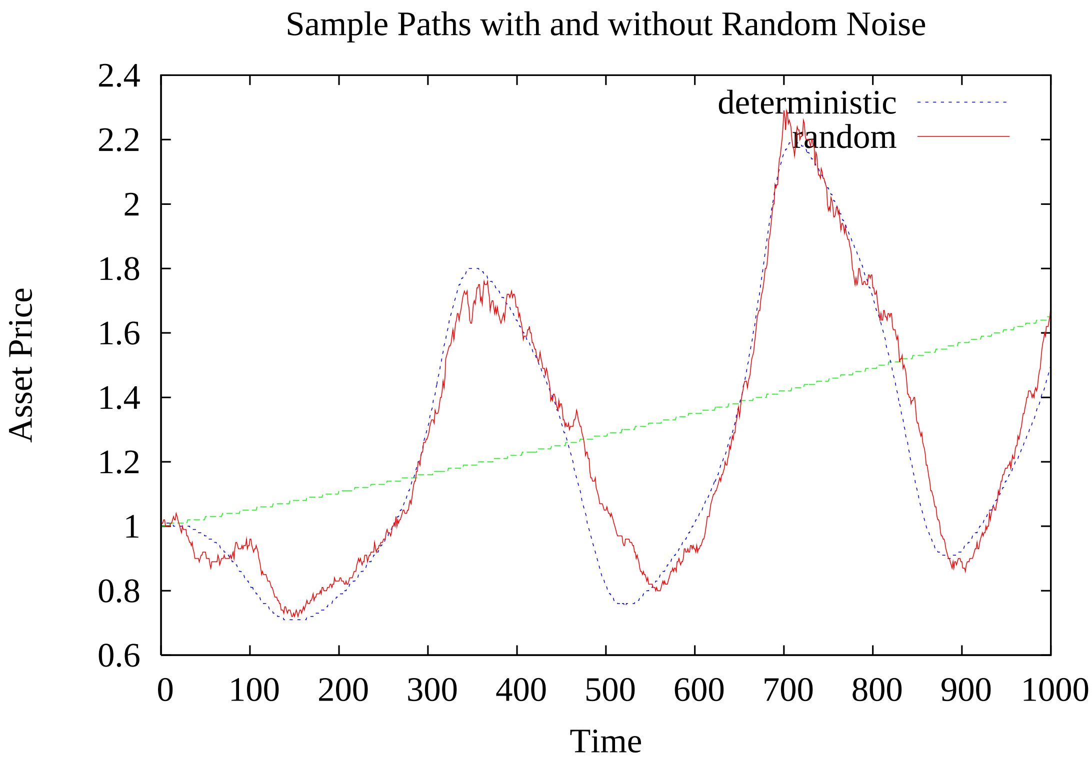

Figure 1: Deterministic and random sample paths of risky asset prices.

Our approach in this study is to consider a simple model of decision makers who use past price data of a risky asset to forecast future prices of the asset, and who base their portfolio allocation decisions on such forecasts. Specifically, we consider decision makers who place greater weight on more recent price data, and who extrapolate past price ratio data in exponential smoothing fashion. This forecast attempts to capture the behavior described nicely by Kindleberger and Aliber (2005, p. 37): "At some stage in the late 1970s the market price of gold was increasing because the market price of gold was increasing. Investors were extrapolating from the increase in the market price from Monday to Tuesday to project the market price on Friday; they purchased gold on Wednesday in anticipation that they could sell at a higher price on Friday." Clearly, this description applies not only to gold speculation, but also to investment in other assets such as real estate. In the model, the extrapolated forecast is modified to bring it closer to the fundamental value, especially if the deviation between the forecast and the fundamental value is large. This naturally leads to "bursting" of bubbles, and to cycles during which asset prices spend most time significantly above or below the fundamental value. The deterministic price trajectories may converge to a fixed point equal to the fundamental value, or may converge to a periodic attractor, or may converge to an uncountable attractor that describe price cycles around the fundamental value. Figure 1 shows a deterministic and a random sample path of prices that result from the model.

References

C. P. Kindleberger and R. Aliber. *Manias, Panics, and Crashes: A History of Financial Crises.* Wiley, Hoboken, NJ, 2005.