Efficient Rare-event Simulation I: Basic Concepts and Techniques

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This is the first of two lectures whose purpose is to introduce and discuss methodology that is aimed at efficiently study rare events in stochastic process using simulation. Consider for instance models such as queueing systems, studied in manufacturing and communications; reservoir processes and time series models, often used in insurance portfolios and financial models respectively and Gaussian random fields, typically used environmental sciences. These types processes come up often in applied probability and stochastic modeling.

Our goal is to develop fast computational tools to study and quantify the extreme behavior of these types of models. These tools are motivated by the need of studying the impact of rare but consequential events such as network failures in communication applications, bankruptcy in the insurance context or extreme weather conditions in environmental sciences. The types of questions that we are interested in addressing in our lectures are the following:

1) How can one efficiently estimate small overflow probabilities in networks with complex design?

2) How does one efficiently compute conditional expectations given extreme events? For instance, in the context of insurance and finance, how to efficiently evaluate the deficit and severity of large losses given their occurrence?

3) How does one efficiently simulate large excursions of a random fields? This is useful, for instance, if one wishes to study the effects of high contamination in populated cities.

4) What do we mean by efficient? How does one characterize good performance of an algorithm?

The first lecture concentrates on basic principles, definitions and motivation. In particular, we shall discuss what makes an algorithm efficient. How is computational complexity quantified in rare event simulation. We will then discuss some basic methods that are often applied in rare event simulation. In particular, we shall review simulation techniques such as importance sampling and conditional Monte Carlo and illustrate their applicability in examples motivated by some of the applications described above.

We will discuss how to take advantage, in a large class of rare event environments, of a "course" description of the conditional distribution of the process given a rare event of interest in order to design an efficient Monte Carlo procedure. Thereby, enhancing such a "course" description in order to effectively compute, via Monte Carlo, to the conditional dynamics of the process given a rare event with arbitrarily high accuracy.

The lecture will finish with some examples and counterexamples illustrating the challenges that arise when studying rare events in processes with complex dependence, such as networks, or special features such as heavy-tails. The lecture will basically be based on the following references.

Asmussen, S. and Glynn, P. (2008) *Stochastic Simulation: Algorithms and Analysis*. Springer-Verlag. New York, NY.

Bucklew, J. (2004) *Introduction to Rare-event Simulation*. Springer-Verlag. New York, NY.