Control Techniques for Complex Networks

Sean Meyn
Department of Electrical and Computer Engineering and the Coordinated Science Laboratory
University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

The goal in these lectures is to describe a control-theoretic approach to regulating complex networks as found in communication systems, supply chains, building systems, and many other applications.

Consider the systems that are controlled very well today by man-made control systems: Examples include the cruise control in most automobiles, airplanes at take-off, as well as in level flight in the face of wind and unfortunate birds, highway systems, and the electric power grid. All of these systems may appear highly complex, but this complexity is reduced through appropriate feedback mechanisms. Moreover, although an airplane is complex, an effective control design can be obtained by consideration of a naive linear model.

In the first of these two lectures we consider two very simple network models - the fluid model, and the controlled random walk (CRW) model. Although the fluid model has little value for prediction, it is a valuable tool in control of networks, just as fluid models are routinely used for control in other applications (e.g. [1]). There are many results that justify this point of view: It is known that stability of a fluid model implies stability of the stochastic model [6], [3], [4], and solutions to dynamic programming equations (i.e., the value functions) for the two models are closely related [7], [9], [10].

A major gap between the two models is the following. An optimal policy is *myopic* with respect to the solution to the associated dynamic programming equation. Frequently policies are proposed that are myopic with respect to another function such as the cost function itself. For the fluid model it is known that a myopic policy is *always stabilizing*, provided the function used in convex and monotone [2], [8], while for a stochastic model this absolutely false. The reason for this gap is that a stochastic model is subject to greater constraints than the fluid model - e.g., it is possible to serve an empty buffer in the fluid model. We show how to resolve this gap, and in so doing obtain the celebrated MaxWeight policy of Tassiulus [5], and many extensions for robust control and distributed control of complex networks [11].

Given so much insight regarding the structure of value functions, it is natural to apply ideas from machine learning to construct policies, and tune them on-line. The second lecture will provide an introduction to TD-learning for value function approximation, based on the final chapter of [10]. Given a parameterized family of functions, this algorithm constructs the *best* approximate value function over this class. Applications to control of networks is in its infancy — some recent results [12] and potential directions for research will be described.

References

[1] B. D. O. Anderson and J. B. Moore. *Optimal Control: Linear Quadratic Methods*. Prentice-Hall, Englewood Cliffs, NJ, 1990.

[2] H. Chen and D. D. Yao. Dynamic scheduling of a multiclass fluid network. *Operations Res.*, 41(6):1104–1115, November-December 1993.

[3] J. G. Dai. On positive Harris recurrence of multiclass queueing networks: a unified approach via fluid limit models. *Ann. Appl. Probab.*, 5(1):49–77, 1995.

[4] J. G. Dai and S. P. Meyn. Stability and convergence of moments for multiclass queueing networks via fluid limit models. *IEEE Trans. Automat. Control*, 40:1889–1904, November 1995.

[5] L. Georgiadis, M. Neely, and L. Tassiulas. *Resource Allocation and Cross Layer Control in Wireless Networks (Foundations and Trends in Networking, V. 1, No. 1)*. Now Publishers Inc., Hanover, MA, USA, 2006.

[6] V. A. Malyšev and M. V. Menʹšikov. Ergodicity, continuity and analyticity of countable Markov chains. *Trudy Moskov. Mat. Obshch.*, 39:3–48, 235, 1979. *Trans. Moscow Math. Soc.*, pp. 1-48, 1981.

[7] S. P. Meyn. Stability and optimization of queueing networks and their fluid models. In *Mathematics of stochastic manufacturing systems (Williamsburg, VA, 1996)*, pages 175–199. Amer. Math. Soc., Providence, RI, 1997.

[8] S. P. Meyn. Sequencing and routing in multiclass queueing networks. Part I: Feedback regulation. *SIAM J. Control Optim.*, 40(3):741–776, 2001.

[9] S. P. Meyn. Workload models for stochastic networks: Value functions and performance evaluation. *IEEE Trans. Automat. Control*, 50(8):1106– 1122, August 2005.

[10] S. P. Meyn. *Control Techniques for Complex Networks*. Cambridge University Press, Cambridge, 2007.

[11] S. P. Meyn. Stability and asymptotic optimality of generalized MaxWeight policies. Submitted 2006. To appear in *SIAM J. Control and Optim.*, 2008.

[12] C.C. Moallemi, S. Kumar, and B. Van Roy. Approximate and data-driven dynamic programming for queueing networks. Submitted for publication., 2006.