Optimal Spectrum Management: Complexity, Duality and Approximation

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Crosstalk interference is a major obstacle to high speed reliable communication involving multiple users. A standard approach to eliminate multiuser interference is 'orthogonal channelization' whereby users share time, frequency or codes on a non-overlapping basis. Although this approach is easy to implement, it can lead to high system overhead and low system utilization. An important alternative to orthogonal channelization is to allow users' transmit power spectra to overlap over the shared spectrum, in a way that best exploits physical channel conditions. Such dynamic spectrum sharing strategy is flexible and can potentially achieve a substantially higher overall throughput. However, when users' transmit power spectra are allowed to overlap, crosstalk will appear, causing multiuser interference in the system. As a result, each user's performance will depend on not only the power spectral density of his own, but also those of other users in the system. Clearly, proper spectrum (and time) management is required for the maximization of the overall system performance. Dynamic spectrum management problem of this type arises in both wireless systems (e.g., cognitive radios [3]) and wireline communication (e.g., digital subscriber line (DSL) system [6]). In both cases, crosstalk is known to be the major source of signal distortion, and judicious management of spectrum among competing users can have a major impact on the overall system performance.

In this research, we build on the previous work [1, 5] by focussing on two theoretical aspects of the dynamic spectrum management problem: duality gap estimation and polynomial time approximation algorithms. Our contributions are two fold.

1. Under a stronger condition of Lipschitz continuity, we strengthen the asymptotic strong duality result of [5] and the duality gap estimates of Aubin and Ekeland [1] by providing an explicit estimate of the duality gap for the discretized DSM problem with general concave system utility functions. In particular, we provide a unified analysis that shows that the

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* Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN. Research supported in part by the U.S. NSF, grant number DMS-0610037 and in part by the USDOD ARMY, grant number W911NF-05-1-0567.

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duality gap vanishes asymptotically at the rate $O(1/\sqrt{N})$ under a Lipschitz continuity assumption on the channel parameters, where N is the size of the uniform discretization of the shared spectrum. Notice that the zero duality for the continuous formulation requires the use of Lebesgue integrals which cannot be well approximated by a Riemann partial sum. This creates technical difficulties in our analysis since the discretized DSM problem is obtained through the Riemann sum approximation of the corresponding Lebesgue integrals in the continuous formulation. Fortunately, even though the integrals can not be well approximated, the two optimization problems are still asymptotically equivalent, and the duality gap for the discretized problem approaches zero at the rate of $O(1/\sqrt{N})$. This rate improves to $O(1/N)$ when channels are frequency flat.

2. Our second contribution is a polynomial time approximation scheme to determine the optimal Frequency Division Multiple Access (FDMA) spectrum sharing strategy. The reason for focussing on FDMA solutions is two fold. First, the recent work [2] has shown that FDMA strategies are sum-rate optimal for scenarios with strong interference. Thus, there is no loss of generality by considering only FDMA solutions in this case. Secondly, when restricted to the class of FDMA strategies, the Lagrangian dual relaxation can be implemented in polynomial time [4]. Without imposing the FDMA structure, the Lagrangian dual is difficult to compute or optimize. Notice that, due to a positive duality gap, the optimal dual solution does not lead to a feasible FDMA solution with equal utility value for the discretized DSM problem. In this work, we devise a linear programming procedure which, when coupled with the Lagrangian dual relaxation scheme, can generate a near-optimal FDMA solution for the continuous formulation of the DSM problem. By the duality gap estimate $O(1/\sqrt{N})$, we show that this combined procedure constitutes a fully polynomial time approximation scheme for the continuous version of the DSM problem.

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