

Approximating Submodular Functions

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Algorithms that involve submodular functions usually assume that they are given by an evaluation oracle. In particular, submodular functions can be minimized by using only polynomially many queries to the oracle.

In this talk, we consider the problem of approximating a nonnegative, monotone, submodular function f on a ground set V of cardinality n *everywhere*, after only $\text{poly}(n)$ oracle queries. The main result is a deterministic algorithm that makes $\text{poly}(n)$ oracle queries and derives a function \hat{f} such that $\hat{f}(S) \leq f(S) \leq \alpha(n)\hat{f}(S)$ holds for every subset $S \subseteq V$, where $\alpha(n) = \sqrt{n+1}$ for rank functions of matroids and $\alpha(n) = O(\sqrt{n} \log n)$ for general monotone submodular functions. This result is based on approximately finding a maximum volume inscribed ellipsoid in asymmetrized polymatroid. The analysis uses the greedy algorithm for approximately maximizing a monotone submodular function subject to a cardinality constraints by Nemhauser, Wolsey, and Fischer [2].

This is joint work with Michel Goemans, Nick Harvey, and Vahab Mirrokni [1].

References

- [1] M. X. Goemans, N. J. A. Harvey, S. Iwata and V. Mirrokni: Approximating submodular functions everywhere, *Proceedings of the 20th Annual ACM-SIAM Symposium on Discrete Algorithms* (2009), to appear.
- [2] G. L. Nemhauser, L. A. Wolsey, and M. L. Fisher: An analysis of approximations for maximizing submodular set functions I, *Math. Programming*, 14 (1978), 265–294.