Minimizing Submodular Functions

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A function f defined on the subsets of a finite set V is *submodular* if it satisfies

$$f(X) + f(Y) \geq f(X \cap Y) + f(X \cup Y), \qquad \forall X, Y \subseteq V.$$

Submodular functions are discrete analogues of convex functions [5]. Examples include cut capacity functions, matroid rank functions, and entropy functions.

The first polynomial algorithm for submodular function minimization by Grötschel, Lovász, and Schrijver [1] is based on the ellipsoid method. Recently, combinatorial polynomial algorithms have been developed [3, 7], and the current best weakly and strongly polynomial bounds [2, 6] are $O((n^4\mathrm{EO} + n^5)\log M)$ and $O(n^5\mathrm{EO} + n^6)$, where EO is the time for function evaluation, n is the cardinality of the ground set V and M is the maximum absolute value of the function values.

In this talk, I will review algorithms and applications of minimizing submodular functions. In particular, I will present a new combinatorial algorithm obtained in recent joint work with Jim Orlin [4].

References

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