Stochastic Combinatorial Optimization
II: Extensions and Online Problems

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The field of stochastic optimization goes back at least 50 years, with the classical work of Dantzig and Beale, and a rich flourishing body of work. The study of this area through the viewglass of approximation algorithms, however, is somewhat more recent, starting with the work on Dye, Stougie, and Tomasgard [1], Immorlica, Karger, Minkoff and Mirrokni [5], and of Ravi & Sinha [6], all appearing within the past 4 years. This line of work recognized that stochastic optimization often involved computatinally hard problems to be solved (at times because the underlying problems were hard, and at other times because the stochastic optimization gave it a layer of hardness), and hence it made sense to also give provable bounds on the performance of polynomial-time heuristics for these problems. There has been much work since then in this area, and these two lectures will attempt to give an overview of some of the ideas, techniques and results.

Extensions of the Two Stage Stochastic Model. In the two-stage model discussed in the previous lecture, there were two stages of decision-making: the *anticipatory* or first stage, and the *recourse* or second stage. But one may imagine the information about the final demand set as being revealed over many stages, with the possibility of taking some corrective action at each stage, but where the costs increase as more information becomes available. Approximation algorithms have been given for these multi-stage optimization problems as well: see, e.g., [7, 3, 4], and we will discuss some of the attendant issues.

The natural limiting case of the multi-stage model is the infinite horizon case, which can be thought of as *stochastic online optimization*. Again, we consider some combinatorial optimization problem: at each time step we get a new demand drawn from some known distribution. What elements should we buy today given that we do not know the precise future (and just know the probability distribution it is drawn from)? We will discuss some recent results [2] for these problems, and try to place it in the bigger context.

Finally, if we have time, we will discuss other variants of the stochastic model, as well as results in the demand-robust framework.

References

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