Stochastic Combinatorial Optimization
I: Two Stage Optimization with Recourse

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The field of stochastic optimization goes back at least 50 years, with the classical work of Dantzig and Beale, and a rich flourishing body of work. The study of this area through the viewglass of approximation algorithms, however, is somewhat more recent, starting with the work on Dye, Stougie, and Tomasgard [2], Immorlica, Karger, Minkoff and Mirrokni [6], and of Ravi & Sinha [7], all appearing within the past 4 years. This line of work recognized that stochastic optimization often involved computatinally hard problems to be solved (at times because the underlying problems were hard, and at other times because the stochastic optimization gave it a layer of hardness), and hence it made sense to also give provable bounds on the performance of polynomial-time heuristics for these problems. There has been much work since then in this area, and these two lectures will attempt to give an overview of some of the ideas, techniques and results.

Two Stage Optimization with Recourse. Consider a combinatorial optimization problem which choose elements to minimize the total cost of constructing a feasible solution that satisfies requirements of clients: e.g., choosing a set of edges to connect up the client terminals in the *Steiner tree* problem. We consider a stochastic version of such a problem where the solution is constructed in two stages: Before the actual requirements materialize, we can choose some elements in a *first* stage. The actual requirements are then revealed, drawn from a pre-specified probability distribution π; thereupon, some more elements may be chosen to obtain a feasible solution for the actual requirements. However, in this *second* (recourse) stage, choosing an element is potentially costlier. The goal is to minimize the *expected total cost*: that is, the first stage cost plus the expected second stage cost.

We will discuss the two currently used approaches to solve these problems in the two-stage framework: those that have involved solving linear programming relaxations of these stochastic optimization problems, and then rounding them (e.g., as in [7, 8, 5] and others that take "well-behaved" existing algorithms for the underlying combinatorial optimization problem and use them to solve their stochastic versions (e.g., as in [4, 3]). We will also discuss several of the modeling issues involved, including how the probability distribution π is represented and the associated question of scenario reduction (see, e.g., [1]), and how different models for cost inflation in the second stage lead to differing levels of intractability.

References

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