

On the correlation structure of a Lévy-driven queue

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Abstract

In this paper we consider a single-server queue with Lévy input, and in particular its workload process $(Q_t)_{t \geq 0}$, with a focus on the correlation structure. With the correlation function defined as $r(t) := \text{Cov}(Q_0, Q_t) / \text{Var } Q_0$ (assuming that the workload process is in stationarity at time 0), we first determine its transform $\int_0^\infty r(t)e^{-\vartheta t} dt$. This expression allows us to prove that $r(\cdot)$ is positive, decreasing, and convex, relying on the machinery of completely monotone functions. We also show that $r(\cdot)$ can be represented as the complementary distribution function of a specific random variable. These results are used to compute the asymptotics of $r(t)$, for t large, for the cases of light-tailed and heavy-tailed Lévy input.

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