Conic Optimization, Randomization, and Combinatorial Problems

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In this lecture we shall discuss the applications of conic optimization and semidefinite programming (SDP) in the design of randomized algorithms for solving discrete optimization problems. Such problems arise from a great variety of applications, including graph theory, signal processing, portfolio selection, and so on. We shall discuss in particular the use of complex variables in this approach. The entire lecture will be divided into three parts. The first part includes a general introduction to: complex SDP, quadratic modeling, SDP relaxation, randomization algorithms, rounding procedures based on normal distributions ([1]). Then, in the second part we shall show how to apply these techniques to derive approximation bounds (or ratios) for quadratic models, using complex decision variables ([2]). In the third and last part of the lecture, we shall discuss applications of the randomization algorithms in portfolio selection. In particular, we consider the situation where there is a constraint on the cardinality of the assets being selected in the portfolio. Alternatively, the investor may decide to hold each asset either with a substantial amount or not at all, in order to increase the efficiency of the portfolio management. In both cases, the decision models, though practical and appealing, are exceedingly hard combinatorial problems that are notoriously difficult to solve to optimality. We shall showcase the power of relaxation and randomization through analyzing its effects on these particular applications.

References

[1] S. Zhang and Y.W. Huang. Complex Quadratic Optimization and Semidefinite Programming. *SIAM Journal on Optimization* 16, 871 – 890, 2006.

[2] Z.Q. Luo, N.D. Sidiropoulos, P. Tseng, and S. Zhang. Approximation Bounds for Quadratic Optimization with Homogeneous Quadratic Constraints. To appear in *SIAM Journal on Optimization*.