Planning of buses and gates at Amsterdam Airport Schiphol

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Overview

Introduction column generation

- Solving large linear programming problems

Gate planning

- Introduction
- Model
- Solving

Bus planning

- Introduction
- Model
- Solving

Integrating the two problems

Introduction column generation

Column generation

$$\begin{aligned} \min\ & cx \\ \text{s.t.}\quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

- Problem: Number of variables too big to consider explicitly
- In optimal solution, most variables will have value 0
- Only subset of all variables is interesting

Column generation

$$\min \; cx$$

s.t.

$$Ax \leq b$$

$$x \geq 0$$

- Problem: Number of variables too big to consider explicitly
- In optimal solution, most variables will have value 0
- Only subset of all variables is interesting

Column generation will only consider variables that have potential to decrease objective.

Variable can decrease objective if reduced cost are negative.

Column generation (2)

Split the problem into two problems:

Master problem	**Sub problem**

Column generation (2)

Split the problem into two problems:

Master problem

$$\min cx'$$

s.t.

$$Ax' \leq b$$

$$x' \geq 0$$

where $x' \subseteq x$

Other variables are zero by definition

Sub problem

Column generation (2)

Split the problem into two problems:

Master problem

$$\min\ cx'$$

s.t.

$$Ax' \leq b$$

$$x' \geq 0$$

where $x' \subseteq x$

Other variables are zero by definition

Sub problem

- Determine new variable to add to master problem
- Do not consider all variables explicitly, but find variable with *minimum* reduced cost
- If min reduced cost < 0
  - Add variable to x'
  - Resolve master problem
- If min reduced cost $\geq 0 \rightarrow$ finished

Gate planning

Problem description

We have a set of flights:

- Arrival and departure time
- Type of aircraft
- Region of origin/destination (Schengen/EU/Non-EU)
- Preferences of airline
- Ground handler

And we have a set of gates

- Possible regions (Schengen/EU/Non-EU)
- Possible aircraft
- Possible ground handlers

Problem description (2)

Goal:

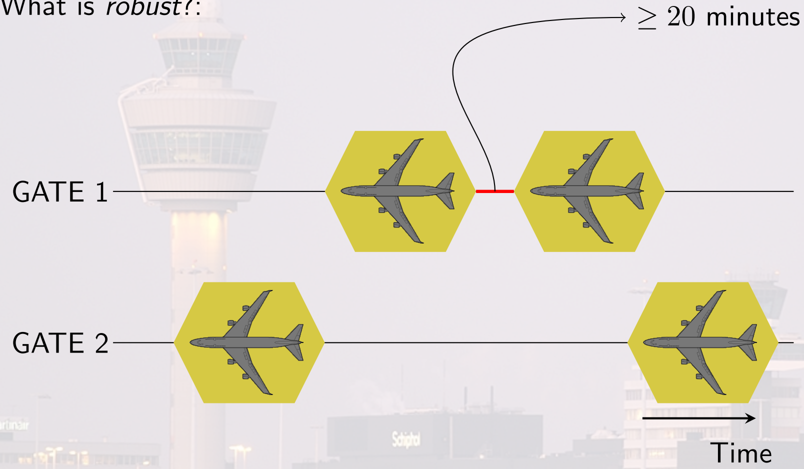
- find assignment that is as *robust* as possible

that satisfies:

- region constraints
- aircraft constraints
- ground handler constraints
- time constraints

Problem description (3)

What is *robust*?:



Problem description (3)

What is *robust*?:

Problem description (4)

Cost function:

- High for small separation times
- Low for long separation times
- Descending steeply in beginning

Refinements:

- Certain combinations of flights are more desirable
- Certain assignments are less desirable

Gate plans

Distinguish only between gate types (not between individual gates)

Gate plan:

- Set of flights assigned to the same gate
- Designed for a given type of gate
- Cost of gate plan = cost due to corresponding separation times

We can incorporate all mentioned constraints within *valid* gate plans

The model

$$\text{Min.} \sum_{i=1}^{N} c_i x_i \qquad \text{s.t.}$$

$$\begin{aligned}
\sum_{i=1}^{N} g_{vi} x_i &= 1 \quad \text{for } v = 1, \ldots, V \\
\sum_{i=1}^{N} e_{ia} x_i &= S_a \quad \text{for } a = 1, \ldots, A \\
\sum_{i=1}^{N} \sum_{v=1}^{V} \sum_{a=1}^{A} p_{vak} e_{ia} g_{vi} x_i &\geq P_k \quad \text{for } k = 1, \ldots, K \\
x_i \in \{0, 1\} & \qquad i = 1, \ldots, N
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$$\text{Min. } \sum_{i=1}^{N} c_i x_i + \sum_{v=1}^{V} Q_v \mathrm{UAF}_v \quad \text{s.t.}$$

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\end{aligned}$$

Solving

We use column generation to solve this problem.

For each gate type a we want to find a new gate plan with minimum reduced cost.

Create graph G_a:

- Vertex for every possible flight v
- Arc (v, v') if flight v' can be placed after flight v
- Set cost arc (v, v') to contribution flight v to reduced cost

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Observations:

- Path $\leftrightarrow$ Gate plan
- Path cost $\rightarrow$ Reduced cost of gate plan

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Solving (2)

Solution might be fractional → Convert to ILP

Enhancement of column generation

Creating extra columns during column generation:

- Solving pricing problem resulted in shortest path
- Disable flights from this new gate plan one by one and solve shortest path again

Add the unique columns to the ILP problem and solve it:

- Speeds up the ILP solving tremendously
- Gives better solutions: $Z_{LP} \leq Z_{GA} \leq Z_{ILP'} \leq Z_{ILP}$

Conclusion gate planning

- Second phase: assign gate plans to gates
- Fast enough (some minutes for solving complete day)
- Small integrality gap
- Additional feature:
  - Automatic splitting of flights

Bus planning

Problem description

- Some stands don't have air bridge
- These passengers need to be transported via buses
  - Transporting busload of passengers to/from plane we call *Trip*
- Lot of similarities with Gate planning:

Flight	$\longrightarrow$	Trip
Gate type	$\longrightarrow$	Shift
Gate plan	$\longrightarrow$	Bus plan

- Differences:
  - Bus drivers must get some breaks during shift
  - There are two types of buses

The model

$$\text{Min.} \sum_{j=1}^{M} c_j y_j + \sum_{t=1}^{T} R_t \text{UAT}_t \quad \text{s.t.}$$

$$\begin{aligned}
\text{UAT}_t + \sum_{j=1}^{M} h_{tj} y_j &= 1 \qquad t = 1 \ldots T \\
\sum_{j=1}^{M} f_{jb} y_j &\leq T_b \qquad b = 1 \ldots B \\
0 \leq y_j \leq 1 & \qquad\qquad i = 1, \ldots, M
\end{aligned}$$

Solving

Pricing problem similar to gate planning:

- Difference: Some shifts have mandatory break
- Given trip t_1 and t_2. If break in between possible:
  - Add break vertex BV_{t_1,t_2}
  - Add arc $(t_1,\text{BV}_{t_1,t_2})$ with same cost as arc (t_1, t_2)
  - Add arc $(\text{BV}_{t_1,t_2}, t_2)$ with cost 0.
- With minor modifications to the algorithm we will find shortest path including exactly one break vertex.

Integrating the two problems

Integrating the two problems

Some advantages:

- Possibility of feed-back from bus planning to gate planning
- Better overall robustness
- Reducing number of buses needed

Some problems:

- Deal with trips that need not be driven
- Problem becomes considerably larger

The model

Min

s.t.

The model

$$\text{Min} \quad \sum_{i=1}^{N} c_i x_i + \sum_{v=1}^{V} Q_v \text{UAF}_v$$

s.t.

$$\begin{array}{rcll}
\text{UAF}_v + \sum_{i=1}^{N} g_{vi} x_i & = & 1 & v = 1 \dots V \\
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Solving

Solving

Currently working on this...


Questions?