

Initial Transient Problem for Steady-state Output Analysis

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A central problem in the planning of steady-state simulations concerns the issue of how to deal with the presence of the so-called “initial transient”. In particular, suppose that $X(t)$ is the state of the simulation at time t (so that $X(t)$ includes full information on the physical state of the system at time t , as well as the full event schedule at that time). Assume that $X = (X(t) : t \geq 0)$ in an S -valued process that possesses a steady-state, in the sense that

$$X(t) \Rightarrow X(\infty) \tag{1}$$

as $t \rightarrow \infty$, where \Rightarrow denotes weak convergence. For a given performance measure $f : S \rightarrow R$ of interest (such as, for example, the function f that sends a simulation state $x \in S$ into the total workload $f(x)$ associated with state x), the goal of a steady-state simulation is to compute the steady-state expectation

$$\alpha = Ef(X(\infty)).$$

Since the distribution of $X(\infty)$ is unknown, one typically initializes the simulation of X via a distribution for $X(0)$ that is not characteristic of steady-state behavior. For example, production, inventory, and queueing simulations are often initialized so that at time $t = 0$, the content of the system (whether measured in units of inventory, workpieces, or customers) is zero.

As a consequence, the initial segment of the simulation is generally not representative of steady-state behavior. This non-representative initial portion of the simulation is called the “initial transient”. A closely related concept is that of “initial bias”. In particular, it is common (in the presence of assumption (1)) that a strong law of large numbers for $(f(X(t)) : t \geq 0)$ will hold, so that

$$\frac{1}{t} \int_0^t f(X(s)) ds \rightarrow \alpha \quad a.s. \tag{2}$$

as $t \rightarrow \infty$. Given the law of large number (2), the time average estimator $\alpha(t) = t^{-1} \int_0^t f(X(s)) ds$ is the natural estimator for α based on having simulated X to time t . Because of the presence of the initial transient,

$$E\alpha(t) \neq \alpha.$$

The bias $E\alpha(t) - \alpha$ is known as “initial bias.”

This talk is concerned with discussing the state-of-art for both the initial transient and initial bias problems. Among the key issues to be addressed are:

1. What are the types of simulations in which initial transient and initial bias are often particularly troublesome?
2. Are there any theoretical approximations that shed light on the duration of the initial transient?

3. Are there any statistical tools that can be used as initial transient diagnostic?
4. How does the presence of long-range dependence affect the design of steady-state simulations (in view of initial transient)?
5. What, if anything, changes in the setting of multiple parallel processors?

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