Rare–event Simulation via State–dependent Importance Sampling

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The computation of rare–event probabilities arises in many different applications contexts. It is well known that crude Monte Carlo is highly inefficient as a means of computing such quantities. A good alternative is to use importance sampling instead. The key to a successful application of the method is the determination of a good importance distribution. We present a unified approach, capable of dealing with both "light–tailed" and "heavy–tailed" models, for developing such importance distributions. It turns out that good changes–of–measure generally involve state–dependent transitions, even when the original model of interest is built up from state–independent iid sequences. We will show how such state–dependent importance distributions can be used to successfully create asymptotically optimal simulation algorithms both in the light–tailed setting in which conventional large deviations applies and in the context of the heavy–tailed single–server queue.

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