Matchings, Colourings, Dimers

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Abstract

Finding a perfect matching in a bipartite graph belongs to the classical problems in combinatorial optimization and it applies to the optimum assignment problem. We describe a simple linear-time method to find a perfect matching in a regular bipartite graph. The method gives faster algorithms for the problem of colouring the edges of a bipartite graph, another classical problem in combinatorial optimization, which applies to scheduling and timetabling.

The method also gives new methods to find lower bounds on the number of perfect matchings in regular bipartite graphs. It solves a problem of Erdős and Rényi from 1968 and it also gives an improved lower bound for Ising's 'dimer constant'.

In the lecture we will will explain these problems and results.

References

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