Stochastic Networks with Concurrent Resource Occupancy

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Concurrent resource occupancy pervades most engineering and service systems. For example, a multi-leg plane trip requires seat reservation on several connecting flights; a configure-to-order product demands the simultaneous processing of all its components; a long-distance phone call or the streaming of a video on the Internet needs circuit or bandwidth on all the links along its route from source to destination. The object of our study is a class of stochastic networks where jobs require concurrent occupancy of resources, a distinct feature from standard queueing networks ([1]).

Our goal is to do revenue optimization in the network through two interleaved decisions:
(a) *pricing*: to determine the price charged to each job class and its dynamic adjustment over time;
(b) *resource control*: to regulate the distribution of resources among the job classes, in particular, when to accept/reject a job and from which class.

We highlight a new *fixed-point approximation* for a network operating under a set of thresholds that control the access of jobs from each class. In contrast to the fixed-point approximation on the blocking probabilities in prior works ([2, 3]), our fixed-point scheme approximates directly the mean number of accepted orders. It does not rely on the independence among the links, and numerical studies have shown that its accuracy is consistently very good. With this fixed-point approximation, the resource control problem takes the form of setting the optimal thresholds, which can be formulated and solved as a linear program. The LP duality points an optimal solution that takes the form of a critical ratio associated with the demand distribution, reminiscent of the newsvendor model. Furthermore, we can show that our approach based on the fixed-point approximation is optimal in some asymptotic sense.

References

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