The Stochastic Knapsack and Dynamic Pricing

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The stochastic knapsack originated from a model that studies admission control in telecommunications [5, 6]. In recent years, a variation of the model has become a basic tool in studying problems that arise in revenue management and dynamic pricing; and it is in this context that our study is undertaken. Based on a dynamic programming (DP) formulation and associated properties of the value function, we identify a lower- and upper-orthant structure of the optimal policy in accepting/rejecting orders — with prices given. We show that this structure is a consequence of familiar properties in related queueing control problems, such as monotonicity, concavity, and submodularity and its variations; refer to [1, 2, 3, 4].

Although the DP can be efficiently solved, it does not result in a tractable value function, upon which pricing optimization can be carried out. The structure of the DP problem, however, motivates a class of control that we call switch-over policies — start from accepting only orders of the highest price, and switch to including lower prices as time goes by, with the switch-over times optimally derived via convex programming. Although the switch-over policy is suboptimal in general, we show it is asymptotically optimal when key parameters grow. The switch-over policy also results in a very tractable value function suitable for pricing optimization. We develop pricing models based on this policy to optimize the price reductions over the decision horizon.

References

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