Decentralized Optimization, Stochastic-Process Limits, and System Dynamics

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We consider a mathematical framework for decentralized optimization and dynamic optimal control in stochastic systems that are motivated by recent trends of autonomic systems arising in various applications areas; e.g., refer to [3] for applications in computing systems. While a decentralized approach for optimization is natural in large-scale autonomic systems due to inherent overheads and delays, a centralized approach with complete knowledge over all constituent system components has the potential to provide significant improvements over a decentralized approach, in the same way that solutions to global optimization problems (if attainable) are often superior to the corresponding locally optimal solutions.

We first establish conditions under which decentralized optimization can provide the same quality of solution as centralized optimization within the context of autonomic stochastic systems. Our study also considers algorithmic issues where additional information is passed between system components at different levels of a hierarchical decentralized optimization model in order to significantly increase the efficiency with which the optimal solution is computed. As a specific instance of decentralized optimization, we investigate the fundamental problem of allocating servers among multiple customer environments and routing customers among the distributed parallel queues within each environment to minimize an objective function based on equilibrium sojourn times. Optimal solutions for this stochastic system under general assumptions for the arrival and service processes are derived through stochastic-process limits.

We then exploit our mathematical framework to investigate the dynamic control properties of decentralized optimization, as such decisions must be made continually over time and at multiple time scales. When the input parameters to the stochastic system change with time, or when the stochastic system changes over time for any other reason, the additional dynamical behavior of the resulting continual optimization problem can be quite complex and a fundamental problem is to determine this interaction between dynamics and optimization. Our analysis illustrates some of these potential difficulties and added complexities of decentralized optimization under uncertainties in the ever changing information that is communicated among parts of the autonomic system over time, including phase transitions, chaos and instability [2].

These issues are investigated within the context of the stochastic system that optimally allocates servers among multiple customer environments and optimally routes customers among the distributed parallel queues within each environment, both continually over time and at multiple time scales.

References

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