

Some Interesting New Perspectives on Tandem Jackson Networks

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Abstract

The study of queueing networks can be traced back to a 1957 paper by J.R. Jackson [2]. Jackson's Theorem gave conditions for a network of queues to have a product form stationary distribution. Even though the theorem is almost fifty years old, it still has its doubters, as illustrated by a recent paper of Li and Li [1]. Despite the efforts of these authors, there is no reason to doubt Jackson's Theorem. However, there are still new and interesting facts to discover about Jackson networks.

In this talk, I shall tell the story of Jackson's Theorem and offer some suggestions as to why it has been widely misunderstood over the years. Following this, I shall present some new results concerning the simplest possible Jackson network: the two-node tandem. It turns out that, by formulating this model in a new way, as a quasi-birth-and-death process with an infinite phase space, we can establish that the rate of decay of the stationary distribution as the number of customers at the second queue becomes large has several interesting properties that have not been observed before.

In particular, we show that it is possible to obtain any decay rate from a certain range by controlling only the transition structure when the second queue is empty. Moreover, if we consider the sequence of tandem queues that is constructed by restricting the waiting room of the first queue to some finite value, and then allowing this value to increase to infinity, the decay rates for the finite truncations converge to a value, which is not necessarily the decay rate in the infinite waiting room case. This work is reported in [3].

References

- [1] Li G. and Li V.O.K. (2003). Networks of Queues: Myth and Reality, *Technical Report, Department of Electrical and Electronic Engineering, The University of Hong Kong.*
- [2] Jackson J.R. (1957). Networks of Waiting Lines, *Operations Research* **5**, 518-521.
- [3] Kroese D.P., Scheinhardt W.R.W. and Taylor P.G. (2003). *Spectral Properties of the Tandem Jackson Network, seen as a Quasi-Birth-and-Death Process*, to appear in *Annals of Applied Probability*.