On the k-Splittable Flow Problem

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In traditional multi-commodity flow theory, the task is to send a certain amount of each commodity from its start to its target node, subject to capacity constraints on the edges. However, no restriction is imposed on the number of paths used for delivering each commodity; it is thus feasible to spread the flow over a large number of different paths. Motivated by routing problems arising in real-life applications, e.g., telecommunication, unsplittable flows have moved into the focus of research. Here, the demand of each commodity may not be split but has to be sent along a single path; see, e.g., (Kleinberg 1996; Dinitz, Garg, and Goemans 1999; Kolliopoulos and Stein 2002; Skutella 2002).

Baier, Köhler, and Skutella (2003) study a generalization of this problem. In the considered flow model, a commodity can be split into a bounded number of chunks which can then be routed on different paths. Restrictions of this kind might occur, for instance, in transportation logistics: A number of different commodities has to be delivered to various destinations by means of trains (or ships). The possible train connections define a network with capacities on the arcs. In order to convey a commodity, it has to be packed into special containers which can then be loaded on trains. In this context, it seems to be natural to impose bounds on the number of containers available for each commodity. Speaking in flow terminology, we thus have to bound the number of paths used for delivering a commodity.

In contrast to classical (splittable) flows and unsplittable flows, already the single-commodity case of this problem is NP-hard and even hard to approximate. On the other hand, approximation algorithms for the single- and multi-commodity case can be obtained; for the latter case, these algorithms are based on strong connections to unsplittable flows. Moreover, there is a variant of the famous maximum-flow minimum-cut theorem for k-splittable flows.

References

Baier, G., E. Köhler, and M. Skutella (2003). On the k-splittable flow problem. *Algorithmica*. To appear.

Dinitz, Y., N. Garg, and M. X. Goemans (1999). On the single source unsplittable flow problem. *Combinatorica 19*, 17–41.

Kleinberg, J. M. (1996). *Approximation algorithms for disjoint paths problems*. Ph. D. thesis, M.I.T.

Kolliopoulos, S. G. and C. Stein (2002). Approximation algorithms for single-source unsplittable flow. *SIAM Journal on Computing 31*, 919–946.

Skutella, M. (2002). Approximating the single source unsplittable min-cost flow problem. *Mathematical Programming 91*, 493–514.