Flows Over Time: Algorithms & Complexity

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A crucial characteristic of network flows occuring in real-world applications is flow variation over time. This characteristic is not captured by classical 'static' network flow models known from the literature. Moreover, apart from the effect that flow values on arcs may change over time, there is a second temporal dimension in many applications: Usually, flow does not travel instantaneously through a network but requires a certain amount of time to travel through arcs.

Ford and Fulkerson (1958) introduce the notion of *flows over time* (also called 'dynamic flows') which comprises both temporal features mentioned above. They consider networks with capacities and transit times on the arcs and give an efficient algorithm for the problem of sending the maximum possible amount of flow from a source to a sink within a given time horizon. In order to get an intuitive understanding of flows over time, one can associate arcs of the network with pipes in a pipeline system for transporting some kind of fluid. The length of each pipeline determines the transit time of the corresponding arc while the width determines its capacity.

Surprisingly, Klinz and Woeginger (1995) show that the problem of computing a minimum cost flow over time is NP-hard. On the other hand, this problem can be solved in pseudo-polynomial time by a static min-cost flow computation in a so-called time-expanded network. The same observation holds for multicommodity flows over time which have only recently been shown to be NP-hard (Hall, Hippler, and Skutella 2003). Various approximation results for these NP-hard problems have been developed by Fleischer and Skutella (2002, 2003).

The intention of the lecture is to give an introduction into the area of flows over time. We will also discuss some of the more recent approximation results just mentioned.

References

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