Fluid Model With Jumps

Joint work with Ivo Adan and Elena Tzenova.

A stochastic fluid queuing system describes the input-output flow of a fluid in a storage device, called a buffer. The rates at which the fluid enters and leaves the buffer depend on a random environment process that is assumed to be an irreducible CTMC. Fluid flow models provide an important tool for the performance analysis of high-speed data networks, or large-scale production systems where a large number of relatively small jobs are processed. Fluid flow models are also used as models of the asymptotic behavior of queues in heavy traffic. Most of the classical research on stochastic fluid systems in the area of telecommunications is based on the work of Anick, Mitra and Sondhi [1] which is an extension of the pioneering work of Kosten [2]. Also see the survey paper by Kulkarni [3] for an extensive overview of the research in this area.

In this presentation we consider a modification of the classical fluid model as follows: the buffer content process increases continuously at a rate depending on the state of the environment and can also have instantaneous upward jumps. A similar stochastic model with downward jumps is studied in Sengupta [4].

Let $X(t)$ be the buffer content and $I(t)$ be the state of the external environment at time t. $\{I(t), t \geq 0\}$ is assumed to be a stochastic process with a finite state space $S = \{1, \ldots, N\}$. While $I(t) = i$ the buffer content increases continuously with rate $r_i \in (-\infty, \infty)$. The environment stays in state $i \in S$ for an exponential amount of time with mean $1/q_i$ and then jumps to state $j \in S$ (not necessarily different from i) with probability $p_{i,j}$. When the $I(t)$ process jumps from state i to state j the amount of fluid in the buffer can increase by a lumpsum *non-negative* random amount with a given c.d.f. $G_{ij}(y),\ y \geq 0$.

We study the bivariate process $\{(X(t), I(t)), t \geq 0\}$. We derive its stability condition, and compute the limiting distributions

$$F_i(x) := \lim_{t\to\infty} P(X(t) \leq x, I(t) = i),\ x \geq 0,\ i \in S.$$

explicitly in terms of the solutions of a generalized eigen-value problem. The theory is applied to several well-known queueing problems.

References

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[4] Sengupta, B. (1989) Markov processes whose steady state distribution is matrix-exponential with an application to the $GI/PH/1$ queue. it Adv. in Appl. Prob. 21, pp 159-180.