The algorithmic infrastructure for the surrogate management framework *

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Abstract

In the second lecture, we will give numerical results for the surrogate management framework applied to some engineering design problems. The present algorithmic infrastructure for the surrogate management framework is based on the GPS algorithm class defined by Torczon [8]. Reorganizing this class of algorithms in [4] has allowed us to solve engineering design problems that were unapproachable by standard optimization algorithms. After that, in [2] we gave a nonsmooth analysis of the GPS class of algorithms, and this analysis pointed directly to the major difficulty with the GPS algorithm. This difficulty is the restriction of the GPS class to finitely many directions.

This talk introduces the Mesh Adaptive Direct Search (MADS) class of algorithms that will form the basis for the next generation of the surrogate management framework. MADS extends the Generalized Pattern Search (GPS) class of algorithms by allowing local exploration of the space of variables in a dense set of directions. As a consequence, MADS can treat any constraints that satisfy a reasonable constraint qualification due to Rockafeller [7] by setting the objective to infinity for infeasible points and treating the problem as unconstrained. For GPS, this could only be done if all the tangent cone generators were generated by finitely many vectors, which were included in the predefined set of allowable directions. Furthermore, MADS guarantees stronger optimality conditions to hold at certain limit points $\hat{x}$ produced by the algorithm even in the unconstrained case. Our main result is that if the objective function f is Lipschitz near $\hat{x}$, and if the hypertangent cone to the feasible region at $\hat{x}$ is nonempty, then the Clarke generalized derivative at $\hat{x}$ is nonnegative in the Clarke tangent cone: *i.e.,* $\hat{x}$ is a

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Clarke stationary point [5, 6]. A corollary to this result is that if f is strictly differentiable at $\hat{x}$ and if the feasible region is regular at $\hat{x}$, then $\hat{x}$ is a contingent KKT stationary point. Another corollary in the unconstrained case is that zero is contained in the generalized gradient of f at $\hat{x}$. These results do not hold for GPS [1]

We present implementable instances of this class of algorithm, and illustrate and compare them with GPS on some test problems.

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