Stochastic networks in the presence of heavy tails: details, results, conjectures

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Queues with heavy-tailed input distributions were first explored in the early 1970's by W.L. Smith ([8]) and J.W. Cohen ([3]) in the context of the tail of stationary delay, $P(D > x),\ x \geq 0$ for the single-server queue (under various special conditions such as regularly varying service time distributions, Poisson arrivals.). Such results were then generalized to the FIFO GI/GI/1 queue with *subexponential*[1] service times by A.G. Pakes in 1975 ([5]). The basic finding was that the asymptotic for $P(D > x),\ x \to \infty$ is different from the light-tailed case in which D has finite moment generating function if service time S does, and (under further regularity conditions) is asymptotically exponential.

Not until more recently has the heavy-tailed paradigm been taken seriously enough to broadly warrant its relevance in stochastic networks.[2]

Various extensions are now known to hold for some inputs more general than renewal processes, and for increasingly more complex models. In 1999 an entire volume of the journal *Queueing Systems* ([6]) was devoted to queues with heavy-tailed distributions. Even more recently, in the last two years alone there has been significant progress by a variety of authors. We present here some of these results and discuss related topics such as stability and control issues.

References

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[1] Subexponential distributions are a class of heavy-tailed distributions; they were first introduced in [2] in the context of branching processes.

[2] Parallel to and intimately connected to this has been the development of insurance risk theory with heavy-tailed claim sizes. In this "dual" setting the quantity of interest is the probability of ruin when starting with initial reserve x, $P(\tau(x) < \infty)$, and the regular variation asymptotics were obtained by B. von Bar in 1975 ([1]), and then extended to the general subexponential case by P. Embrechts and N. Veraverbeke in 1982 ([4]).