Cost and benefit sharing: incentives and axiomatics.

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The equitable division of a joint cost (or a jointly produced output) among agents with different shares or types of output (or input) commodities, is a central theme of the theory of cooperative games with transferable utility. Ever since Shapley's seminal contribution in 1953, this question has generated some of the deepest axiomatic results of modern microeconomic theory.

The Additivity axiom requires individual cost shares to depend additively upon the cost function. It is an invariance property with no equity content. The Dummy axiom requires to charge nothing to an agent whose marginal cost is zero at any level of demand by the other agents.

Any deterministic rationing method generates, by simple integration, a cost-sharing method meeting Additivity and Dummy. Conversely, a cost-sharing method meeting these two axioms is a convex combination of the above. This important representation result holds true whether the different commodities are divisible or come in indivisible units.

Proportional rationing thus corresponds to the Aumann-Shapley cost-sharing method. Uniform rationing (fair queuing) to serial cost sharing; rationing by random priority to the Shapley-Shubik cost-sharing method.

We characterize successively each one of these three methods by means of additional axioms. Demand monotonicity states that raising one's demand never causes one's cost share to fall. Unit Invariance requires that cost shares be unchanged when we change the unit in which one of the commodities is measured.

When the cost function is supermodular (decreasing returns to scale), the serial cost-sharing method has uniquely strong incentive compatibility properties.

Two very promising avenues for future research consist of dropping either the Dummy axiom or the Additivity axiom.

References

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This Chapter contains an extensive bibliography relevant to both lectures of Moulin.
[2] H. Moulin and S. Shenker: *"Serial Cost Sharing"*, Econometrica 50, 1009–1039 (1992).
[3] H. Moulin: *"Incremental Cost Sharing: Characterization by Group Strategyproofness",* Social Choice and Welfare 16, 279–320 (1999).