Dividing a commodity according to claims:
markovian, consistent and strategyproof methods.

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Rationing according to claims is the simplest formal model to date in distributive justice. Each concerned agent holds a claim/demand over the resources, but the resources available fall short of the sum of individual claims. The resources are from a single homogeneous good, and "more is better" for every agent (individual preferences increase in the amount of good received).

We discuss the discrete version of this problem, where units are indivisible and randomly allocated for fairness. The canonical example is that of a server processing one job per unit of time, and of agents who each request a different number of jobs: the server chooses an algorithm to randomly schedule the successive jobs.

The proportional method schedules the jobs sequentially and at any step the probability that an agent is served, is proportional to the number of his jobs in the remaining queue.

Fair queuing (Shenker 1989) schedules jobs in successive round-robin fashion, where each agent receives one unit in each round until his demand is met and the ordering is random in each round. Fair queuing*, the reverse scheduling of fair queuing, serves first (with uniform probability) one of the users with the largest remaining demand.

We identify first principles (axioms) from which the three methods above, and many more, can be derived. These axioms include the symmetric treatment of agents for fairness; the Markovian property that the scheduling sequence is history independent; Demand Monotonicity stating that increasing my demand cannot result in increased delay; and Consistency requiring that the relative scheduling of any two agents be independent of other agents' demands.

We also discuss a variant of the model where there is no upperbound on the number of jobs per consumer, that is conveniently modeled as the filling of an urn with balls of different colors. The Polya-Eggenberger methods place in an urn a fixed number of balls and draw from the urn with replacement of two balls of the color drawn. These methods stand out of the axiomatic discussion.

References

[1] H. Moulin: "*The Proportional Random Allocation of Indivisible Units*", Social Choice and Welfare, 19, 381-413 (2002).
[2] H. Moulin and R. Stong: *"Fair Queuing and other Probabilistic Allocation Methods*", Mathematics of Operations Research, 27, 1-31 (2002).
[3] H. Moulin and R. Stong: *"Filling a multicolor urn: an axiomatic analysis*," forthcoming in Games and Economic Behavior.
[4] H. Moulin: *"Axiomatic Cost and Surplus-Sharing",* forthcoming in: Arrow, Sen and Suzumura (eds.) "*The Handbook of Social Choice and Welfare"*, North-Holland (2002).
This Chapter contains an extensive bibliography relevant to both lectures of Moulin.