Solving optimization problems with complementarity constraints

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One attractive alternative to solving Mathematical Programs with Complementarity Constraints (MPCCs) is to employ standard techniques for Nonlinear Programming (NLP) and solve the equivalent NLP

$$\begin{array}{ll} \mathit{minimize} & f(x) \\ \mathit{subectto} & c_{\mathcal{E}}(x) = 0 \\ & c_{\mathcal{I}}(x) \geq 0 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_1^T x_2 \leq 0. \end{array} \tag{1}$$

This approach has the advantage that it allows the use of fast large scale NLP solvers. However, over recent years, it has been argued that since (1) violates a standard stability condition, NLP solvers cannot be used to solve MPCCs. Both numerical and theoretical evidence has been advanced which support this folklore.

This lecture re-examines this assertion. We present some startling numerical evidence [2] which suggests that certain NLP methods perform robustly and efficiently on a wide range of optimization problems with complementarity constraints.

Currently, new theoretical insight is being developed into why NLP solvers are capable of solving complementarity constraints, [3, 1, 4]. These developments are summarized both for Sequential Quadratic Programming and Interior Point type algorithms.

Time permitting, new alternative formulations of complementarity constraints are introduced and their theoretical and practical merits are outlined. One such formulation is $\min(x_1, x_2) = 0$. It is shown that the min-function can be smoothed and that the resulting formulation is *exact* for any value of the smoothing parameter under suitable assumptions.

References

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