Modelling with complementarity constraints

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Complementarity constraints, and more generally variational inequalities, often appear as constraints in optimization problems. In general, such problems can be expressed as

$$\begin{array}{ll} minimize & f(x) \\ subjectto & c_{\mathcal{E}}(x) = 0 \\ & c_{\mathcal{I}}(x) \geq 0 \\ & 0 \leq x_1 \ \perp \ x_2 \geq 0, \end{array} \tag{1}$$

where $\perp$ means that either $x_{1i} = 0$ or $x_{2i} = 0$ for $x = (x_0, x_1, x_2)$ with controls $x_0 \in \mathbb{R}^n$ and states $(x_1, x_2) \in \mathbb{R}^{2p}$. Problem (1) is referred to as a Mathematical Program with Complementarity Constraints (MPCC).

Applications of complementarity constraints are widespread and fast growing, see [1, 3, 4]. They cover very diverse areas such as the design of structures involving friction, elasto-hydrodynamic lubrication, taxation models, transportation network design and the modellng of competition in deregulated electricity markets.

This lecture starts by introducing and reviewing complementarity constraints. The usefulness and elegance of this modelling tool is demonstrated on a variety of examples from engineering and management science. The implementation of MPCC models in AMPL www.ampl.com is briefly discussed using models taken from www.mcs.anl.gov/ leyffer/MacMPEC.

We conclude by giving a critical review of solution approaches to MPCCs. We illustrate some of the difficulties and potential pitfalls for practical solution approaches such as convergence to non-stationary points [2].

References

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[4] Outrata, J., Kocvara, M. and Zowe, J. *Nonsmooth Approach to Optimization Problems with Equilibrium Constraints.* Kluwer Academic Publishers, Dordrecht, 1998.