Presolving in quadratic and linear programming

Philippe L. Toint

University of Namur, Belgium

ABSTRACT

Quadratic programs constitute a very important class of nonlinear problems, not only because they arise in their own right (for instance in portfolio management applications), but also because they form the basic subproblems to be solved at each iteration of powerful constrained optimization techniques such as SQP. The first talk having already presented an introduction to SQP methods, not much time will be spent on motivating this importance in much more detail.

One crucial challenge in this area for today is to solve even larger problems (in millions on variables), including for problems which may not be convex. For problems of that size, it often happens that the problem itself is formulated via a modelling language whose main purpose is clarity of concept, ease of expression and model adaptability, but not necessarily conciseness and minimality. As a consequence, it often happens that the stated quadratic problems are overspecified (redundant), or involve parts that can easily been eliminated, thereby simplifying the problem considerably. The simplification being at the same time in size and number of non-zero elements in the sparse matrices that are involved in its statement.

Presolving covers the various techniques that are used to perform these simplifications in an automated manner. They are based on a cunning exploitation of the structure of the associated optimality conditions. If most of these techniques are indeed applicable both for linear and quadratic programs, some of them are specific to the latter case.

The talk will describe a number of these techniques (with varying level of detail). A brief description of the implementation issues will also be given, as a discussion of how to "undo" the problem transformation in order to translate the solution of the reduced problem into the space of the original variables and constraints.

The results of a rather exhaustive numerical experimentation will be discussed, where the potential of the presolving technique will be assessed on both linear and quadratic problems.

<u>Note:</u>

N. Gould is co-author of this work.