Constructions of Random Probability Distributions

Theodore P. Hill

Georgia Institute of Technology, Atlanta, USA

ABSTRACT

In many problems involving a stochastic component, the underlying probability distributions are not known completely. Perhaps all that is known is that the distribution has support in the unit interval, has a given mean and/or variance, has a monotone density, or is purely atomic. Typical objectives in design problems are to identify an optimal or worst-case distribution for an unknown random variable, or to analyze the average performance of a given design against "typical" random input distributions. In such situations, where explicit analytical solutions are often unavailable, it can be very useful to be able to construct a dense family of random probability distributions satisfying the known constraints. The goal of this survey talk is to describe classical and recent techniques for constructing random probability measures, illustrate these methods with several concrete examples, and mention some of the open problems.

References

[1] Aldous, D.: *Tree-based models for random distribution of mass*, Journal of Statistical Physics **73** (1993) 625 - 641.

[2] Bloomer, L. and T. Hill: *Random probability measures with given mean and variance*, accepted for publication in Journal of Theoretical Probability (2001).

[3] Diaconis, P. and D. Freedman: *Iterated random functions*, SIAM Review **41** (1999) 45 - 76.

[4] Dubins, L. and D. Freedman: *Random distribution functions*, Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability **2** (1967) 183 - 214.

[5] Ferguson, T.: *Prior distributions on spaces of probability measures*, Annals of Statistics **2** (1974) 615 - 629.

[6] Graf, S., D. Mauldin and S. Williams: *Random homeomorphisms*, Advances in Mathematics **60** (1986) 239 - 359.

[7] Hill, T. and M. Monticino: *Constructions of random distributions via sequential barycenters*, Annals of Statistics **26** (1998) 1242 - 1253.

[8] Kraft, C.: *A class of distribution functions which have derivatives*, Journal of Applied Probability **1** (1964) 385 - 388.

[9] Lavine, M.: *Some aspects of Polya tree distributions for statistical modeling*, Annals of Statistics **20** (1992) 1222 - 1235.

[10] Mauldin, D., W. Sudderth and S. Williams: *Polya trees and random distributions*, Annals of Statistics **20** (1992) 1203 - 1221.

[11] Mauldin, D. and Williams, S. : *Reinforced random walks and random distributions*, Proceedings of the American Mathematical Society **110** (1990) 251 - 258.

[12] Mauldin, D. and M. Monticino: *Randomly generated distributions*, Israel Journal of Mathematics **91** (1995) 215 - 237.
[13] Monticino, M.: *Constructing prior distributions with trees of exchangeable processes*, Journal of Statistical Planning and Inference **73** (1998) 113 - 133.
[14] Monticino, M.: *How to construct a random probability measure*, International Statistical Review **69** (2001) 153 - 167.