Recent Applications of Benford's Law

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ABSTRACT

The century-old observation of statistical folklore known as Benford's Law says that in many naturally occurring datasets, the significant digits are not uniformly distributed, as one might expect, but rather tend to follow a specific logarithmic distribution. This "law" is currently seeing widespread applications to detection of tax fraud (by the Dutch government, among others), statistical tests for "reasonableness" or heterogeneity of data, and diagnostic tests for mathematical models. The goal of this survey talk is to briefly mention some of the classical and recent empirical evidence of the law, sketch a few ideas such as scale- and base-invariance which underlie the new mathematical and statistical foundations for the law, and then to describe recent applications, drawbacks and limitations of these applications, and open problems.

References

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