Network connectivity: approximation algorithms via iterative rounding I

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ABSTRACT

Network design problems arise in a variety of settings. The traveling salesman problem is one famous example; another is the Steiner tree problem. This talk focuses on telecommunication network design. The goal is to build a network cheaply that satisfies some set of specified connectivity requirements. The Steiner tree is a special case of all the problems we consider, and hence these are hard problems. We examine heuristics that obtain provably good solutions for these problems.

The type of heuristics we consider use linear programs to give a lower bound, and then round the solutions to the linear programs. Rounding fractional solutions to a linear program is a popular and powerful technique in the field of approximation algorithms. There are usually two critical steps to obtain an approximation guarantee: determining a good method of rounding; and proving that this method obtains the guarantee. We review some rounding techniques and present a simple one that yields a very strong approximation guarantee for survivable network design when connectivity requirements are for edge-disjoint paths. Variants of this simple algorithm has been used in practice as a heuristic with much success. While the algorithm is simple, the proof is non-obvious and elegant.

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