Approximating Ternary Optimization Problems via Complex Semidefinite Programming

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In this talk, I will consider optimization problems in which the decisions can take three possible values. I will propose to use the cube roots of unity, 1, $e^{i2\pi/3}$, and $e^{i4\pi/3}$, to represent such ternary decisions. This is recent joint work with David Williamson. Here the natural relaxation is that of unit vectors in complex space. One can then use an extension of semidefinite programming to complex space to solve the natural relaxation, and use a natural extension of the random hyperplane technique to obtain near-optimal solutions to the problems.

In particular, I will consider the problem of maximizing the total weight of satisfied equations $x_u - x_v \equiv c \pmod 3$ and inequations $x_u - x_v \not\equiv c \pmod 3$, where $x_u \in \{0, 1, 2\}$ for all u. This problem can be used to model the maximum 3-cut problem and a directed variant, the maximum 3-dicut problem For the general problem, we obtain a 0.79373-approximation algorithm. If the instance contains only inequations (as it does for max 3-cut), we obtain a performance guarantee of $\frac{7}{12} + \frac{3}{4\pi^2} \arccos^2(-1/4) \approx 0.83601$. This compares with proven performance guarantees of .800217 for max 3-cut (by Frieze and Jerrum) and $\frac{1}{3} + 10^{-8}$ for the general problem (by Andersson, Engebretson, and Hastad). If time permits, I will actually show that the relaxation and algorithm considered by Andersson et al is completely equivalent to ours.

References

[1] G. Andersson, L. Engebretsen, and J. Hastad. A new way to use semidefinite programming with applications to linear equations mod p. In Proceedings of the 10th Annual ACM-SIAM Symposium on Discrete Algorithms, pages 41–50, 1999.

[2] A. Frieze and M. Jerrum. Improved approximation algorithms for MAX k-CUT and MAX BISECTION. Algorithmica, 18:67–81, 1997.