Semidefinite Programming and Approximation Algorithms

Approximation algorithms are efficient algorithms for optimization problems which may not produce an optimum solution, but are guaranteed to give a solution within a certain prespecified ratio of the optimum. Through major advances in complexity theory, the challenge of designing aproximation algorithms with good performance guarantees is now well recognized.

Semidefinite programing is the class of optimization problems with a linear objective function and linear constraints over the cone of real positive semidefinite matrices. It has received much attention in the last 5 years because of a host of applications in control theory and in discrete optimization and because of the discovery of efficient algorithms (interior-point methods and others).

In the first talk, I will start by giving background material on semidefinite programming. Semidefinite programming is a very nice class of convex programs, and inherits several important properties of linear programming. I will then describe how semidefinite programming can be used to design approximation algorithms for combinatorial optimization problems. The simplest illustrative example I will be using is the joint result with David Williamson on the maximum cut problem, where semidefinite programming together with a random hyperplane technique gives an approximation algorithm guaranteeing a cut of value at least 87.8 percent of the optimum. If time permits, other applications will be discussed.

References:

M.X. Goemans and D.P. Williamson, "Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming", J. ACM, 42, 1115-1145, 1995.

M.X. Goemans, "Semidefinite Programming in Combinatorial Optimization", Mathematical Programming, 79, 143-161, 1997.