Bicoloring of Matrices and Integer Polyhedra

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Bicoloring of matrices and integer polyhedra are known to be closely related. A $\{0, +1, -1\}$ matrix A is *totally unimodular* if $\{x \in R^n \mid x \geq 0, Ax \leq b\}$ is an integer polyhedron for every integral vector b. Ghouila Houri showed that totally unimodular matrices have an "equitable" bicoloring. If a $\{0, 1\}$-matrix A is balanced, the associated packing, covering and partitioning polyhedra are integral polyhedra. Berge showed that the columns of a balanced matrix A can be partitioned in two sets of "colors" so that no row of A with at least two ones is unicolor. Also Hall's condition on the existence of perfect matchins can be interpreted as a bicoloring theorem about a matrix that is the constraint matrix of an integer polyhedron. We survey these results and show some extensions.

References

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