Travel Times and Flows in Stochastic Networks

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Important measures of the quality of a stochastic network are customers waiting times in nodes or network sectors, and the time it takes for a customer to traverse a route in the network (e.g., the time it takes to travel from one sector to another, or the time to visit each node at least once).

This talk focuses on waiting and travel times for Jackson network processes. Included are introductions to Palm probabilities for stationary Markovian systems [1, 7], and Little laws for Markovian and general utility processes [7]. Palm probabilities describe the state of the network conditioned that a customer enters a node, or conditioned on other complicated events that may depend on the future as well as the past (e.g., a customer leaves a node and exits the network without returning to that node). Little laws yield formulas for expected waiting times in network sectors and travel times for a variety of routes.

Using Palm probabilities and the equilibrium distribution of Jackson networks, we characterize the distribution of the travel time of a customer on an overtake-free route [3, 4, 7, 8]. We also derive the expected time for a customer to traverse from one network sector to another. Surprisingly, this follows from an abstract function-space representation of stochastic processes and stopping times on these spaces.

A classical result for a stationary $M/M/1$ queueing system is that the departure process is a Poisson process [2, 5]. Similarly, the departure processes from an open stationary Jackson network are Poisson processes [3, 7]. We present conditions under which departure processes from Whittle networks, and internal flows between nodes are Poisson processes. Such results follow from a necessary and sufficient condition under which a point process functional of Markov process is a Poisson process [6].

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