Equilibrium Behavior of Stochastic Networks

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A stochastic network (or queueing network) is a system in which customers move among stations where they receive services. Examples include:
Computer and Telecommunications Networks: Data packets, read/write transactions, files, or telephone calls move among computers, buffers, operators or switching stations.
The Internet: Queries, e-mail, advertisements, purchase orders, news and zillions of other e-messages move among host computers, PC's, people, and mail-order stores.
Manufacturing Networks: Parts, orders, or material move among work stations, inspection points, automatically guided vehicles or storage areas.
Equipment Maintenance Networks: Parts or subsystems move among usage sites and repair facilities.
Logistics and Supply-Chain Networks: Parts, material, personnel, trucks or equipment move among sources, storage depots and production facilities.
Parallel Simulation and Distributed Processing Systems: Messages, data packets and signals move among buffers and processors.

Typical network issues are: Where are its bottlenecks or major delays? How does one network design compare with another? What are good rules for operating the network (e.g. customer priorities or routings)? What is a least-cost network (e.g. numbers of machines, tools or workers). To address such issues requires an understanding of the behavior of the network in terms of the equilibrium (or stationary) probability distribution of the numbers of units at the nodes. These distributions are used to evaluate a variety of performance measures such as throughputs, expected costs, and percentage of time a node is overloaded. The equilibrium distribution is a basic ingredient for constructing objective functions or constraints used in mathematical programming algorithms to select optimal network designs and protocols.

This talk describes several basic Markov network processes, begining with Jackson networks (where the nodes operate independently) and Whittle networks (where services at nodes may depend on congestion at other nodes); see [3, 5, 6]. We characterize their equilibrium distributions, and describe basic performance parameters such as throughput rates on arcs and at nodes, expected customer waiting times, distribution of numbers of customers in sectors of the network, and expected busy periods for servers. Similar results are presented for multiclass Jackson and Whittle networks with multiple types of customers. Examples include Kelly networks, where services and routes depend on a customer's type, and BCMP networks with customer-dependent services.

The rest of the talk is on multivariate batch birth-death processes, reversible Markov network processes, and space-time Markov-Poisson processes. These processes are generalizations of classical birth-death processes, and their equi-

librium distributions are based on the reversibility property of Markov processes [2, 3], properties of point processes [1], and Markov processes on general spaces [4].

References

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