Large deviation principle for a transient deflected random walk on Z_+^2

Nicolai Popov, University of Leiden, The Netherlands

We consider a transient irreducible, aperiodic and homogeneous Markov chain S_t on the state space Z_+^2 in discrete time. We also assume it to be *face*-homogeneous, that is, within each of the following subsets of states, the jumps have the same distribution:

$$\Lambda^{1,2} = \{(i_1, i_2) \in Z_+^2 \mid i_1 > 0, i_2 > 0\}, \quad \Lambda^0 = \{(0,0)\} \quad ,$$
$$\Lambda^1 = \{(i_1, 0) \in Z_+^2 \mid i_1 > 0\} \ , \quad \Lambda^2 = \{(0, i_2) \in Z_+^2 \mid i_2 > 0\}.$$

In [1] the same Markov chain was considered, under the assumption of ergodicity. It was proved that S_t satisfies the large deviation principle with an action functional $\mathcal{L}_\tau, \tau > 0$, which is defined by four functions $L^{1,2}, L^1, L^2, L^0$ corresponding to the four faces. These functions were explicitly calculated.

Clearly, in the ergodic case $L^0 = 0$, since the probability of being in a sufficiently big but finite subset of Z_+^2 containing the point 0, is almost 1. This is not so in the transient case. We prove that in the transient case

$$L^0 = \max_\gamma \min_\Lambda H^\Lambda(\gamma),$$

with Λ running through the faces, γ a parameter and the functions $H^\Lambda(\gamma)$ explicitly defined by transition probabilities of S_t.

References

[1] Ignatyuk I.A., Malyshev V.A. and Scherbakov V.V. *Boundary Effects in Large Deviation Problems*. 1994.