Simulating rare events in a canonical random walk with heavy tailed increments with applications to queueing and insurance

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EXTENDED ABSTRACT

This paper deals with estimating small tail probabilities of the maximum of a random walk, which has a negative drift, and whose increment distribution is heavy-tailed (subexponential) to the right (it may be heavy-tailed or light-tailed to the left). Applications include estimating steady-state waiting time distribution in a GI/GI/1 queue with heavy-tailed service times, estimating the steady state buffer content distributions of fluid queues, and estimating infinite or finite horizon ruin probabilities in insurance risk processes with heavy-tailed claims. It is well-known that naive simulation is ineffective for estimating small probabilities and special fast simulation techniques like importance sampling, multilevel splitting, etc., have to be used. Though there is a vast amount of literature on the rare event simulation of stochastic ystems/processes where all distributions are light tailed, previous fast simulation techniques for stochastic systems with heavy tailed distributions have been confined to that of estimating the tail distribution of a geometric sum of independent subexponential random variables (this has applications for the M/GI/1 queue and insurance claim processes with Poisson arrivals). One of the reason is that the framework used for conducting the importance sampling in the light-tailed setting, i.e., exponential twisting, is no longer applicable here. We develop an importance sampling based approach where we use a change of measure on the increment distribution, based on a new framework called delayed subexponential twisting.

KEYWORDS:
Simulation analysis methodology, variance reduction, importance sampling, rare event simulation, heavy tailed distributions, subexponential distributions, insurance risk, fluid queues, GI/GI/1 queues.

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