Rare event simulation techniques for models of computer and communication systems

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EXTENDED ABSTRACT

This talk reviews some of the fast simulation techniques used for estimating probabilities of rare events and related quantities in stochastic models of computer and communication systems. It is by no means a complete survey of these rare event simulation techniques. However, an attempt will be made to give some of the basic concepts, intuitions, and algorithms used for different types of stochastic models. The reader is referred to Heidelberger (1995) and Shahabuddin (1995) for recent comprehensive surveys in this area, and the reference list of Boots and Shahabuddin (2000) for some of the later papers in this area.

Estimations of the small probabilities of rare events are required in the design and operation of many engineering systems. Consider the case of a telecommunication network. It is customary to model such systems as network of queues, with each queue having a buffer of finite capacity. Information packets that arrive to a queue when its buffer is full are lost. The rare event of interest may be the event of a packet being lost. Current regulations stipulate that the probability of packet loss should not exceed 10 to the power -9. Or in a reliability model of a space craft computer, we may be interested in estimating the probability of the event that the system fails before a mission time. Naturally, one would want this to be extremely low. The main problem with using standard simulation to estimate such small probabilities is that an extraordinarily large number of events have to be simulated in the model before any samples of the rare event may occur. Hence special simulation techniques are needed to make the events of interest occur more frequently.

In this talk, we will focus on importance sampling as it is currently the most widely used rare event simulation technique for stochastic models of computer and communication systems. In importance sampling, the stochastic model is simulated with a new probability dynamics, that makes the events of interest occur more frequently. The sample value is then adjusted to make the final estimate unbiased. However, choosing any change of measure that makes the event of interest occur frequently is not enough; how it is made to happen more frequently is also very important. For example, an arbitrary change of measure that makes the rare event happen more frequently may give an estimator with an infinite variance. Thus the main problem in importance sampling is to come up with an appropriate change of measure for the rare event simulation problem in hand. Different classes of stochastic models usually use changes of measure that are totally different in nature. In this talk, we will attempt to give the flavor behind importance sampling by taking a few canonical stochastic systems, and describing how one obtains efficient changes of measures and derives their "asymptotic optimality". Asymptotic optimality implies that the growth in simulation effort to achieve a given accuracy, is sub-exponential as the event of interest becomes rarer (given an appropriate parameterization), in contrast to naive-simulation where this growth is exponential.

KEYWORDS:
Simulation analysis methodology, variance reduction, importance sampling, rare event simulation, light-tailed distributions, heavy-tailed distributions, random-walks, insurance risk, fluid queues, GI/GI/1 queues, queueing networks.

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