INTERIOR-POINT METHODS FOR NONLINEAR PROGRAMMING

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Over the past several years an interior-point paradigm has evolved for deriving search directions for linear and quadratic programming. The steps in the derivation can be described briefly as follows:

1. To each inequality constraint add/substract a slack variable thereby making every constraint either a simple nonnegativity constraint or an equality constraint.
2. Introduce an associated family of log-barrier problems, parametrized by a real number $\mu \geq 0$, in which the nonnegativity constraints are replaced with log-barrier terms in the objective function.
3. Write down the first-order optimality conditions for the log-barrier problem.
4. Rewrite some of the first-order optimality conditions so that they treat the primal and the dual symmetrically.
5. Apply Newton's method to the optimality conditions to derive a linear system of equations, the KKT system, for *interior-point search directions*.

With these search directions, one can build an interior-point method as follows:

1. Initialize all variables in such a way that the nonnegative ones are strictly positive.
2. Pick a value for μ and compute search directions along which the algorithm will move to a new point.
3. Shorten the search direction vectors as necessary to maintain strict positivity of every nonnegative variable.
4. Move to the new point and repeat the process from step 2.

We have recently applied this paradigm to create a new algorithm for nonlinear programming, which we have implemented in a code called LOQO.

For nonlinear programming, we have made the following changes to the basic algorithm. First, we have added a merit function to help guide the step shortening. Without such a merit function, it is easy to give examples in which the step lengths are too long and the algorithm fails to converge. The function that we use is the barrier function plus a constant times the L^2 norm of the infeasibility vector.

For nonconvex problems, we add one more modification. We perturb the diagonals of a certain block of the KKT system so that the matrix associated with the so-called dual normal equations is positive definite. This perturbation is necessary to ensure that the search directions proceed toward a local minimum (when minimizing).

Recently we have been experimenting with a predictor-corrector variant of the algorithm. We shall describe this variant too.

Preliminary numerical testing indicates that the method is robust. We shall present numerical comparisons with MINOS, SNOPT, and LANCELOT showing that the method is efficient and has the promise of greatly reducing solution times on at least some classes of models.

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