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Stochastic Programming: Applications, Properties and Computational Challenges

Suvrajeet Sen
SIE Department
University of Arizona, Tucson, AZ 85721

Abstract

Stochastic programming deals with a class of optimization models and algorithms in which some of the data may be subject to significant uncertainty. Such models are appropriate when data evolve over time and decisions need to be made prior to observing the entire data stream. For instance, investment decisions in portfolio planning problems must be implemented before stock performance can be observed. Similarly, utilities must plan power generation before the demand for electricity is realized. Such inherent uncertainty is amplified by technological innovation and market forces. As an example, consider the electric power industry. Deregulation of the electric power market, and the possibility of personal electricity generators (e.g. gas turbines) are some of the causes of uncertainty in the industry. Under these circumstances it pays to develop models in which plans are evaluated against a variety of future scenarios that represent alternative outcomes of data. Such models yield plans that are better able to hedge against losses and catastrophic failures. Because of these properties, stochastic programming models have been developed for a variety of applications, including electric power generation (Murphy et al [1982]), financial planning (Cariño et al [1994]), telecommunications network planning (Sen et al [1994]), and supply chain management (Fisher et al [1997]), to mention a few. The widespread applicability of stochastic programming models has attracted considerable attention from the OR/MS community, resulting in several recent books (Kall and Wallace [1994], Birge and Louveaux [1997], Prékopa [1995]) and survey articles (Birge [1997], Sen and Higle [1999]). Nevertheless, stochastic programming models remain one of the more challenging optimization problems.

This presentation will consist of three parts.

1. Applications of Stochastic Programming. After a brief discussion of applications in power systems, production systems, and telecommunications problems, we will discuss a multistage stochastic programming model for financial planning with currency options. Using simulations, we will demonstrate that in imperfect markets, currency hedging

strategies based on stochastic programming can outperform traditional approaches such as Delta hedging.

2. Mathematical Properties. In this lecture, our focus will be on multistage recourse problems. We will begin by reviewing basic properties such as convexity and differentiability (or lack thereof). We will proceed to a study of multistage dual problems. We show that in contrast to primal solutions, optimum dual solutions can be anticipative. We will provide an economic interpretation of these ideas.

3. Computational methods and challenges. This lecture will begin with an overview of some classes of algorithms for recourse problems. We will discuss both deterministic as well as stochastic methods for these problems. We will also discuss some of the challenges, and open questions that are of current interest.

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