APPLICATIONS OF PROBABILITY TO NONSTOCHASTIC PROBLEMS

Sheldon M. Ross

University of California, Berkeley

Extended Abstract

If it is known that 15 squirrels live in a grove of 52 trees arranged in a circular fashion, what is the smallest number x for which it can be stated that there is a group of x consecutive trees that together house at least 3 squirrels?

Whereas a problem such as the preceding is deterministic in nature, it is sometimes possible to introduce probabilities to obtain useful results. The primary technique has been called, by Erdos and his colleagues, the *probabilistic method*. Its general application is to a problem where one is interested in

$$m = \max_{s \in S} f(s)$$

where S is some specified finite set, and f a function defined on the elements of S. A useful bound can often be obtained by letting X be a randomly chosen element of S and then using that

$$m \geq E[f(X)]$$

We will present applications of this technique to the following well-known problems:

1. The Maximum Weighted Independent Set Problem
2. The Maximum Cut Problem
3. The Maximum Number of Hamiltonian Paths in a Tournament

In each of the preceding we will present the classical bound, as given in *The Probabilistic Method*, by Alon, Erdos, Spencer, as well as simply derived improvements.

Consider a cash flow sequence $\mathbf{a} = (a_0, a_1, \ldots, a_n)$ where a_i is to be received at the end of period i. If r_i is the interest rate for period i, $i = 1, \ldots, n$, then the present value of this sequence is

$$P(\mathbf{a}) = a_0 + \sum_{i=1}^{n} a_i \prod_{j=1}^{i} (1 + r_j)^{-1}$$

We show how to use probability to derive a sufficient condition for $P(\mathbf{a})$ to be positive for any sequence of nonnegative interest rates.

Algorithms for solving deterministic problems that make use of probability are called *randomized algorithms*. While these algorithms usually cannot guarantee optimality, they often can quickly find solutions that, with high probability, are optimal. Two examples of randomized algorithms, one relating to the finding of the minimal cut in a network, and the other to the weighted independent set problem, will be presented.