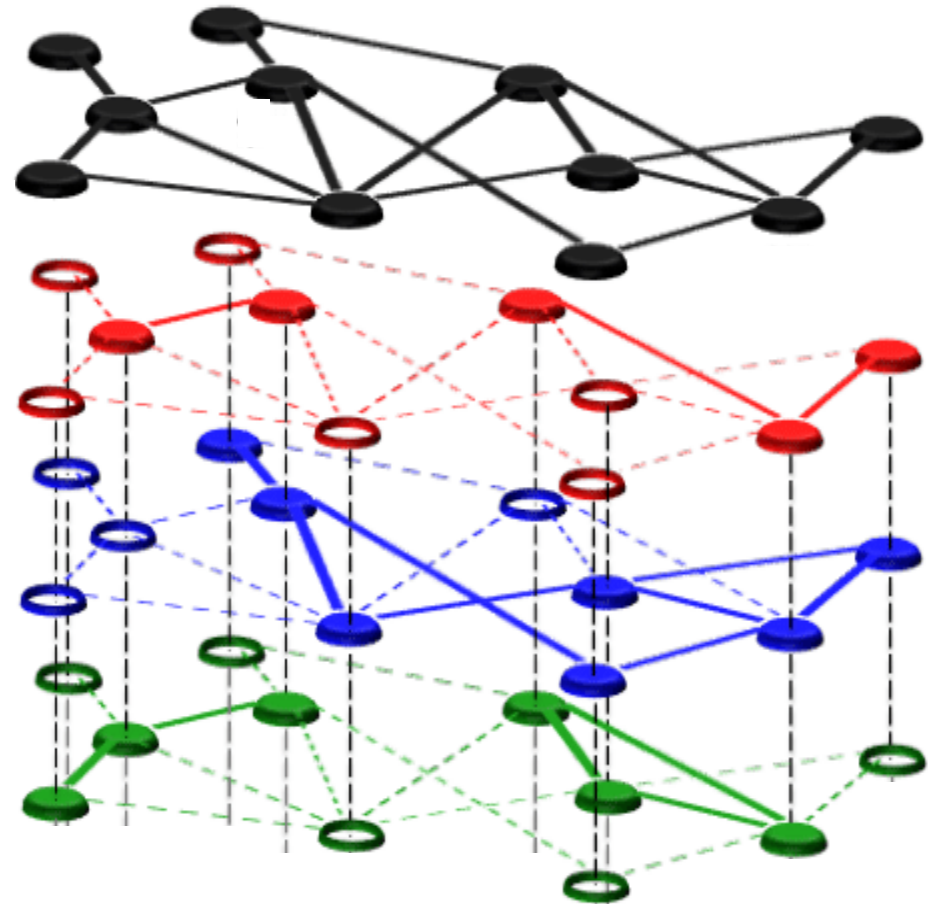
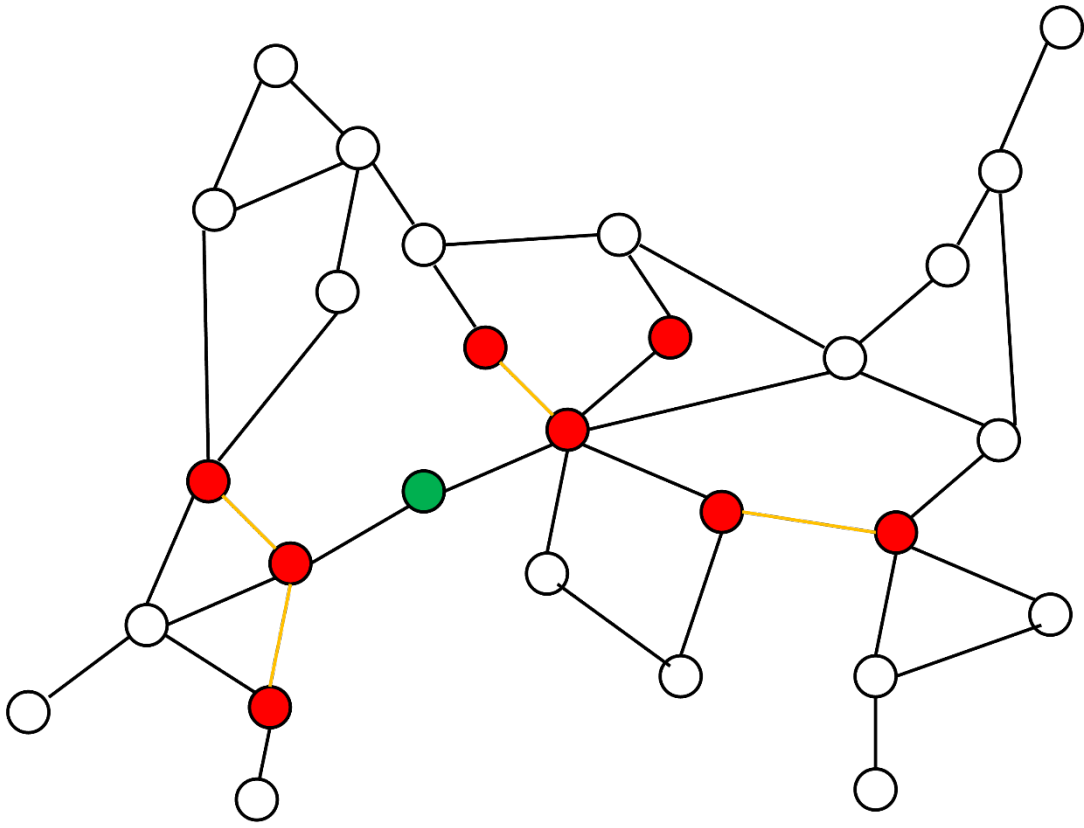
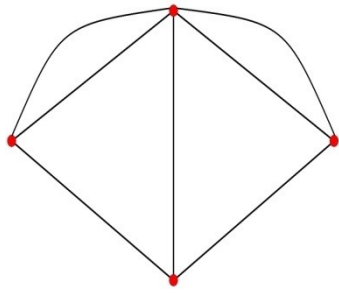


# Multiplexing In Networks and Diffusion



Chandrasekhar, Chaudhary, Golub, Jackson

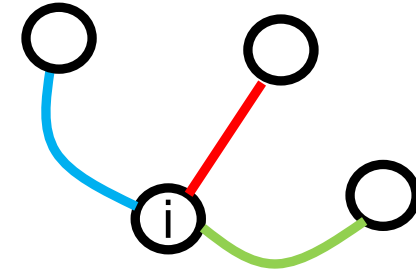
# LNMB



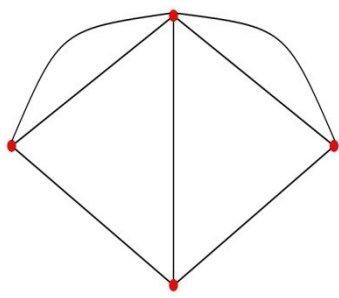
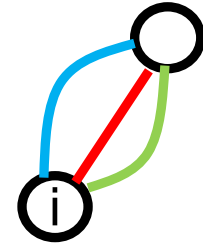
- Data collected to test theory
- Provides new unanticipated empirical effects
- New theoretical questions, requiring new models
- New predictions for the data...

# “Multiplexed” Networks

- Compare two people:
  - One has different friends for different purposes

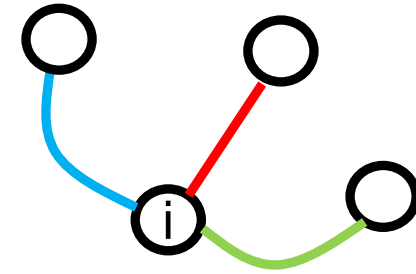


- Another person does everything with the same friend

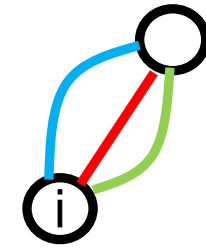


# “Multiplexed” Networks

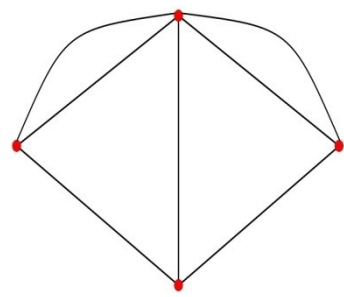
- Compare two people:
  - One has different friends for different purposes



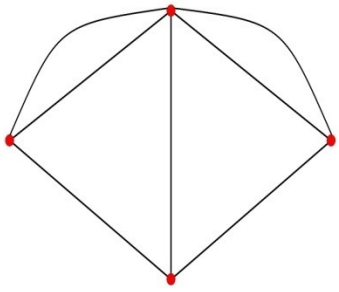
- Another person does everything with the same friend



- Which is more susceptible to contagion?
- Which is more likely to adopt a behavior/new technology?

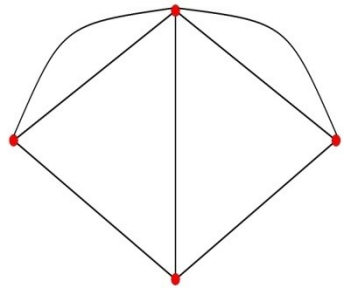


# Outline



- Empirical background multiplexing
- How do multiple layers affect diffusion (in an RCT)?
- Theory on how multiplexing impacts diffusion (simple, complex)

# Background - Microfinance

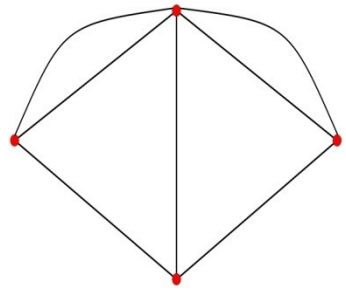


BCDJ 2013, 2019

Dramatically different participation in microfinance across villages

Collected data in Karnataka India 75 villages to test theories of diffusion

# Timeline



- 2006 We surveyed 75 villages that the bank intended to enter
- 2007-2010 Bank entered 43 villages offered loans, not other 32
- 2011-2012 We resurveyed all villages



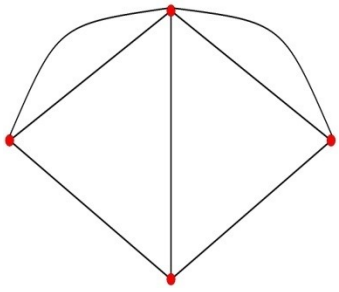
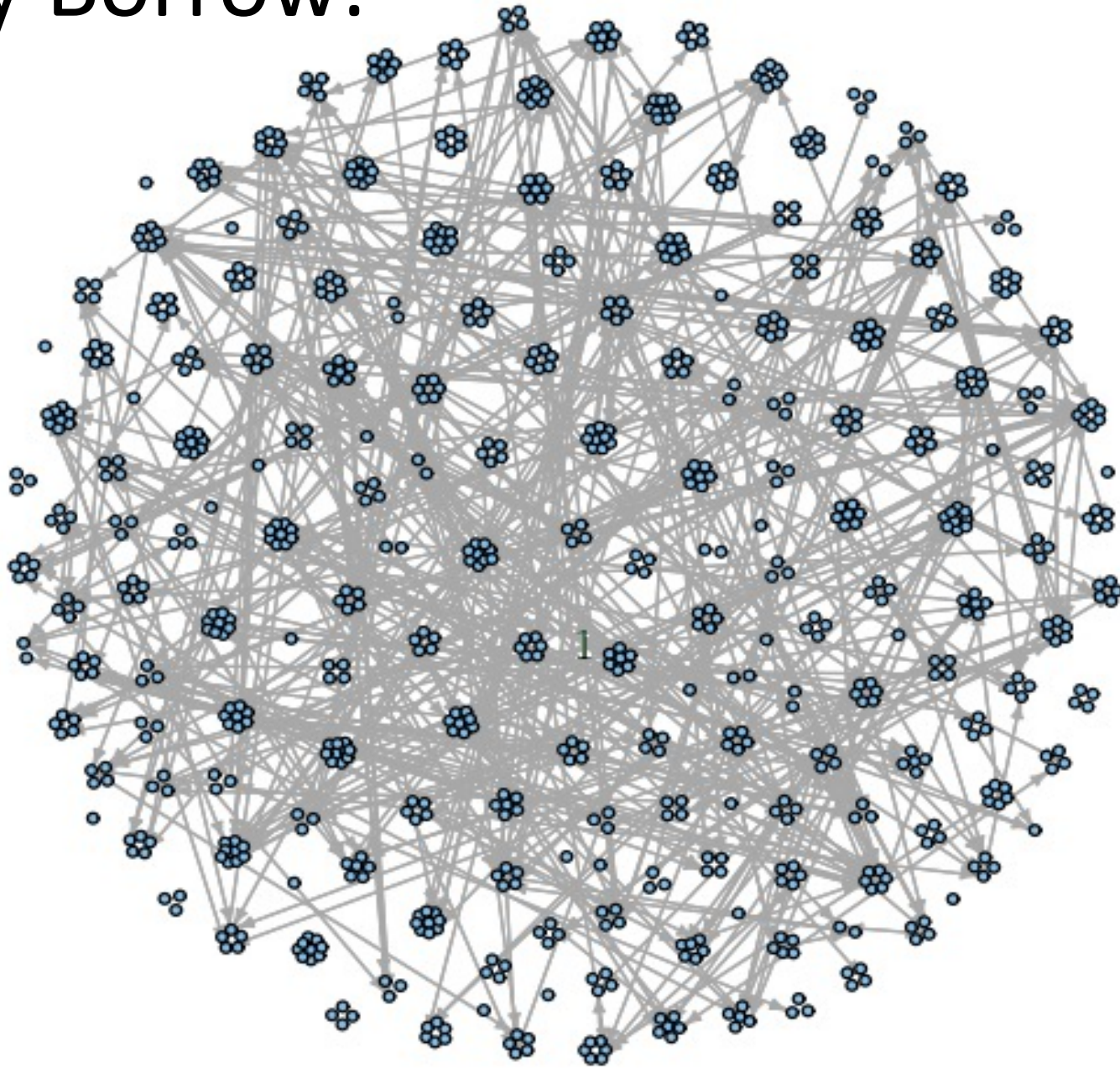
Karnataka



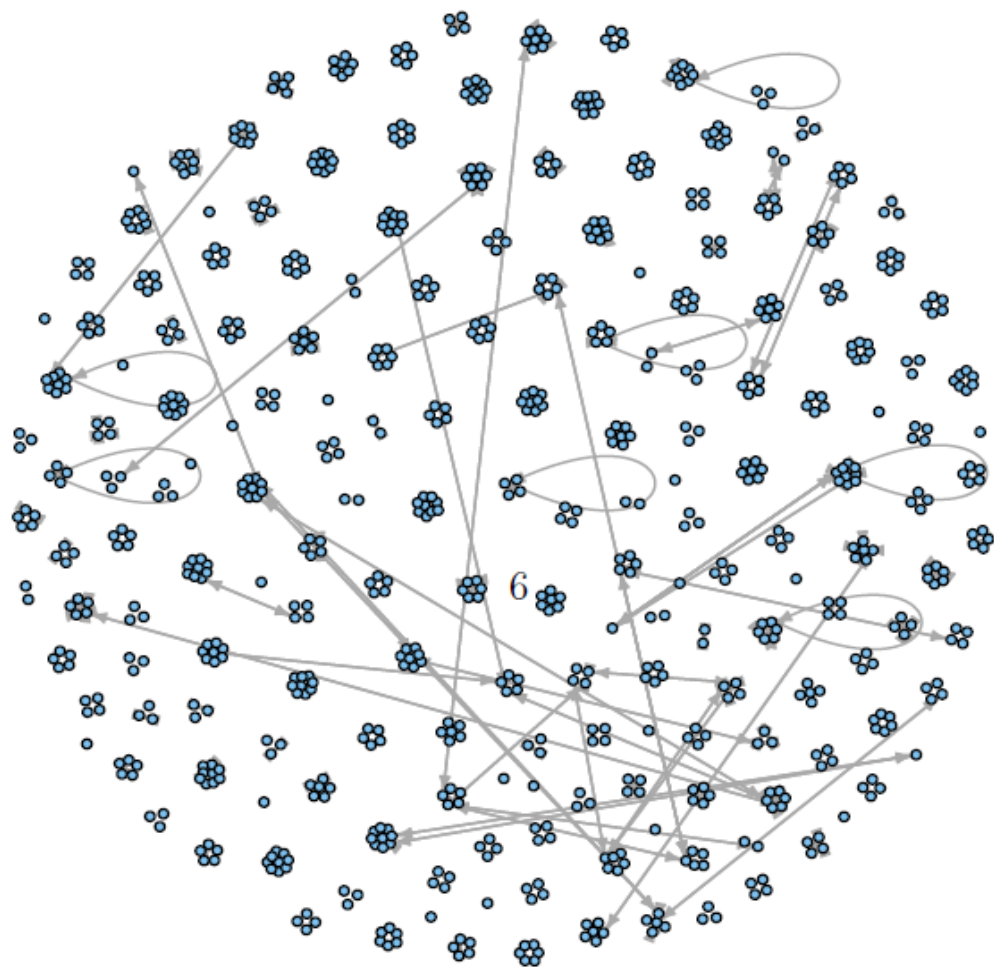




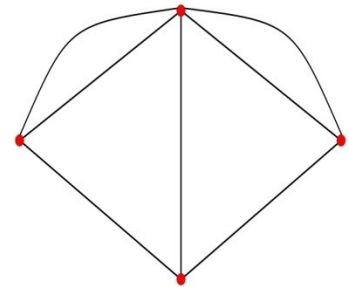
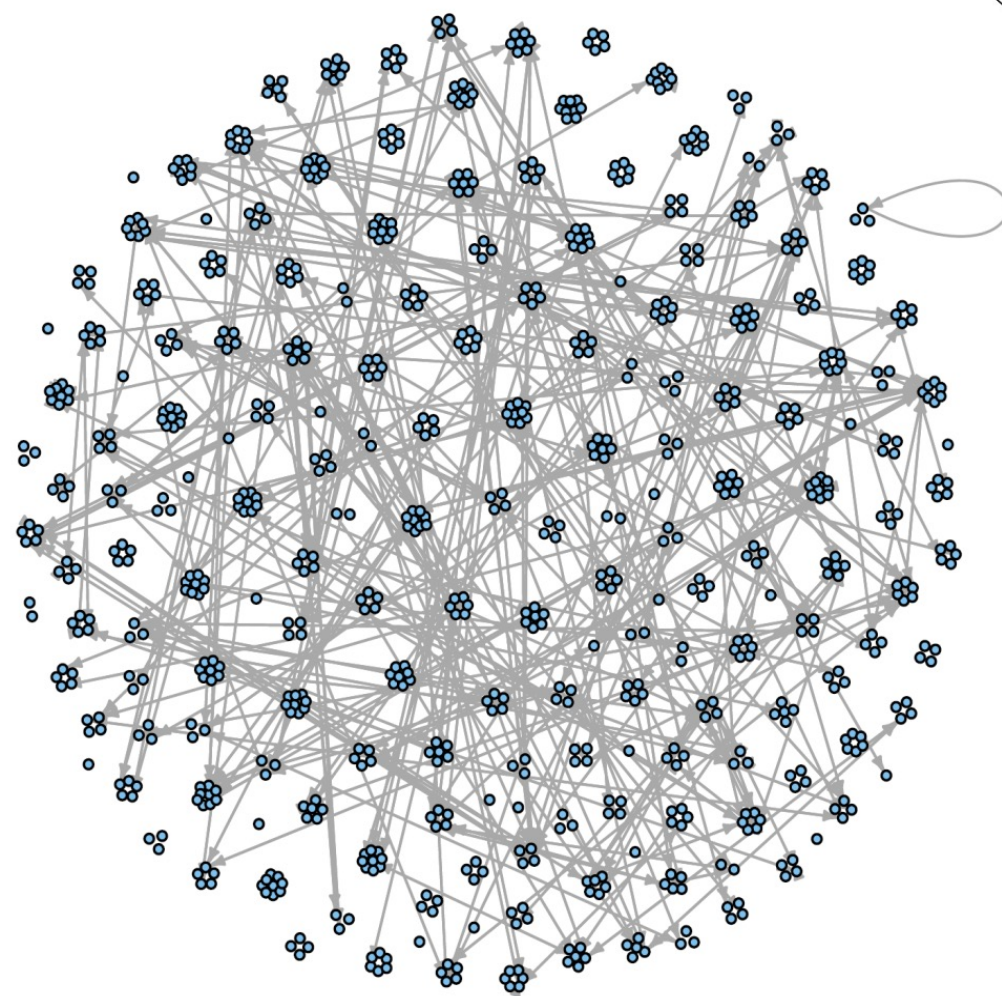
# Money Borrow:



Temple

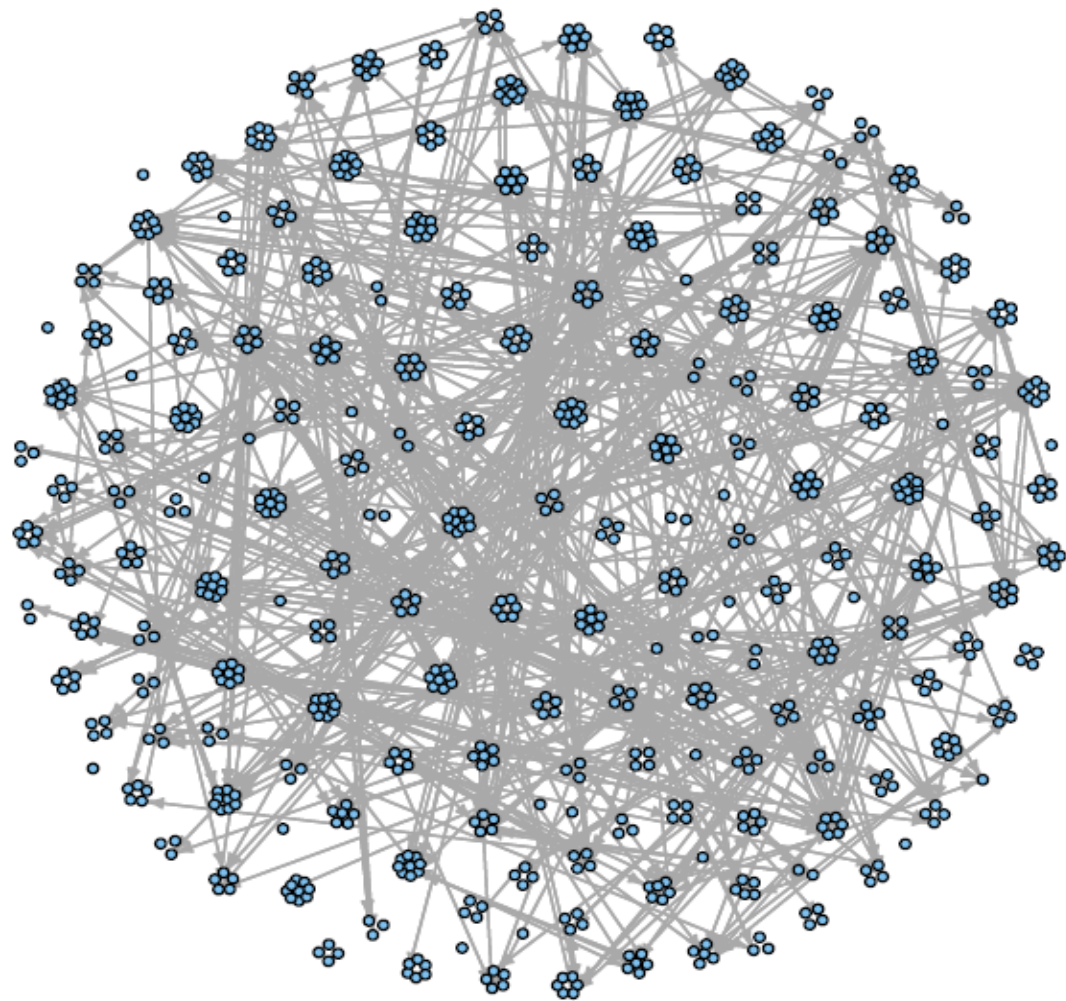


Advice

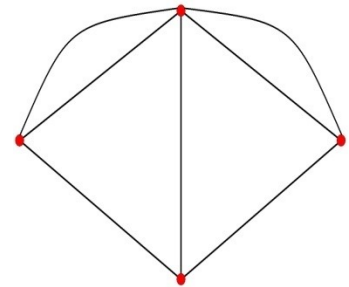
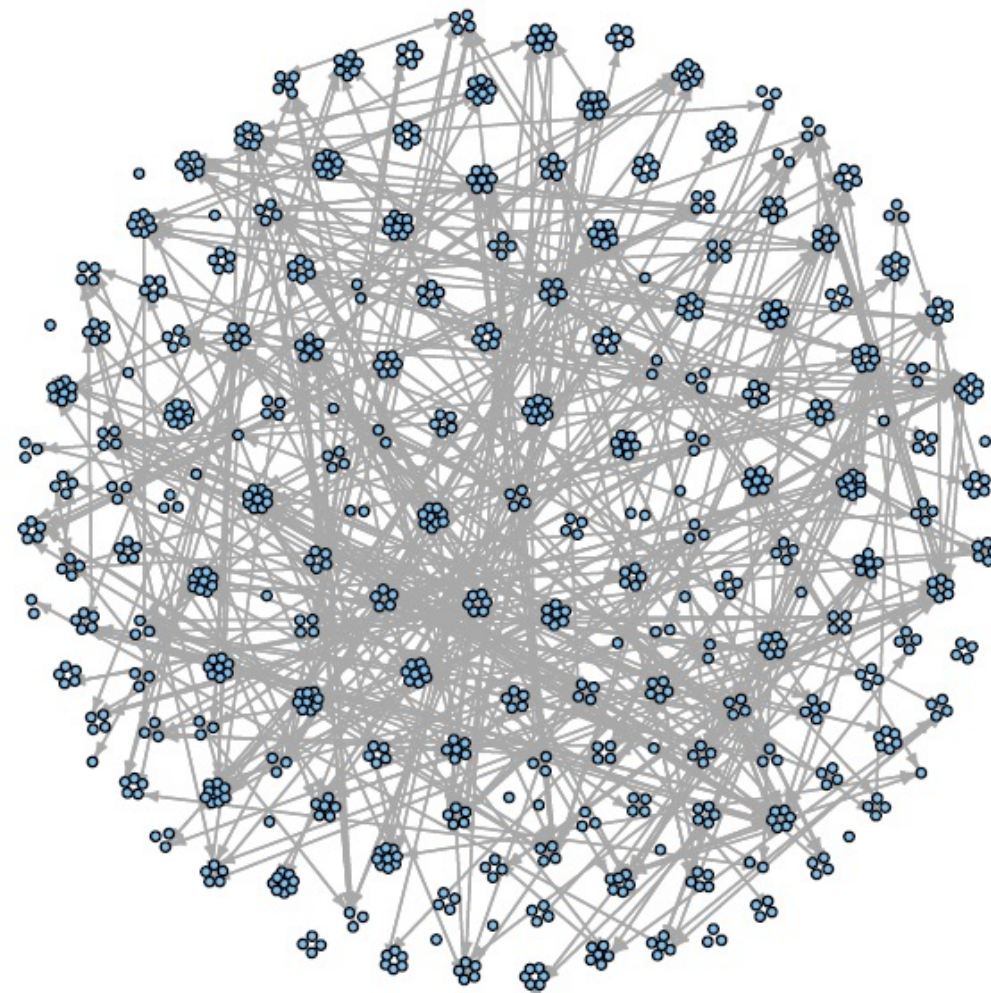




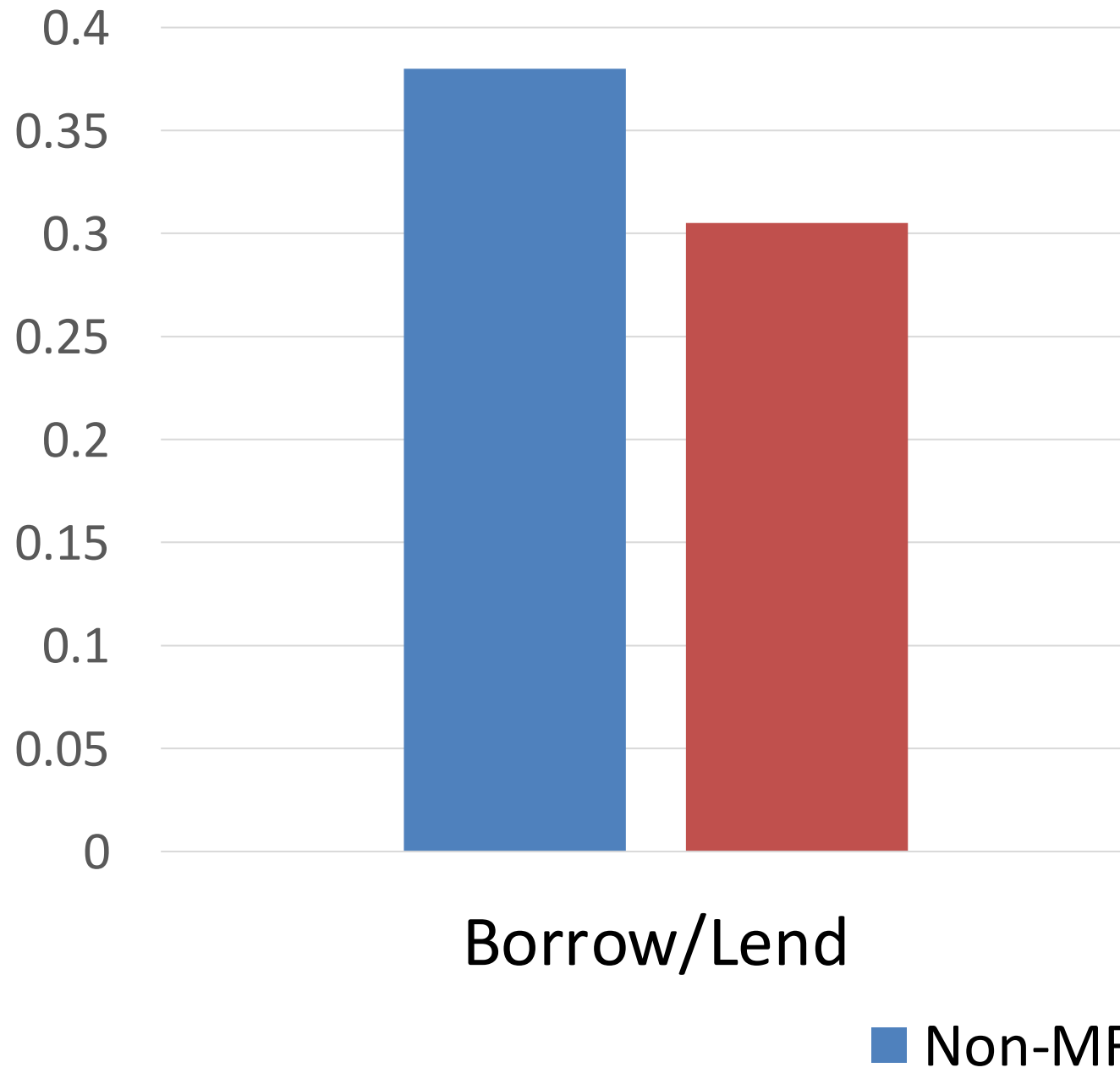
**Kero-Come**



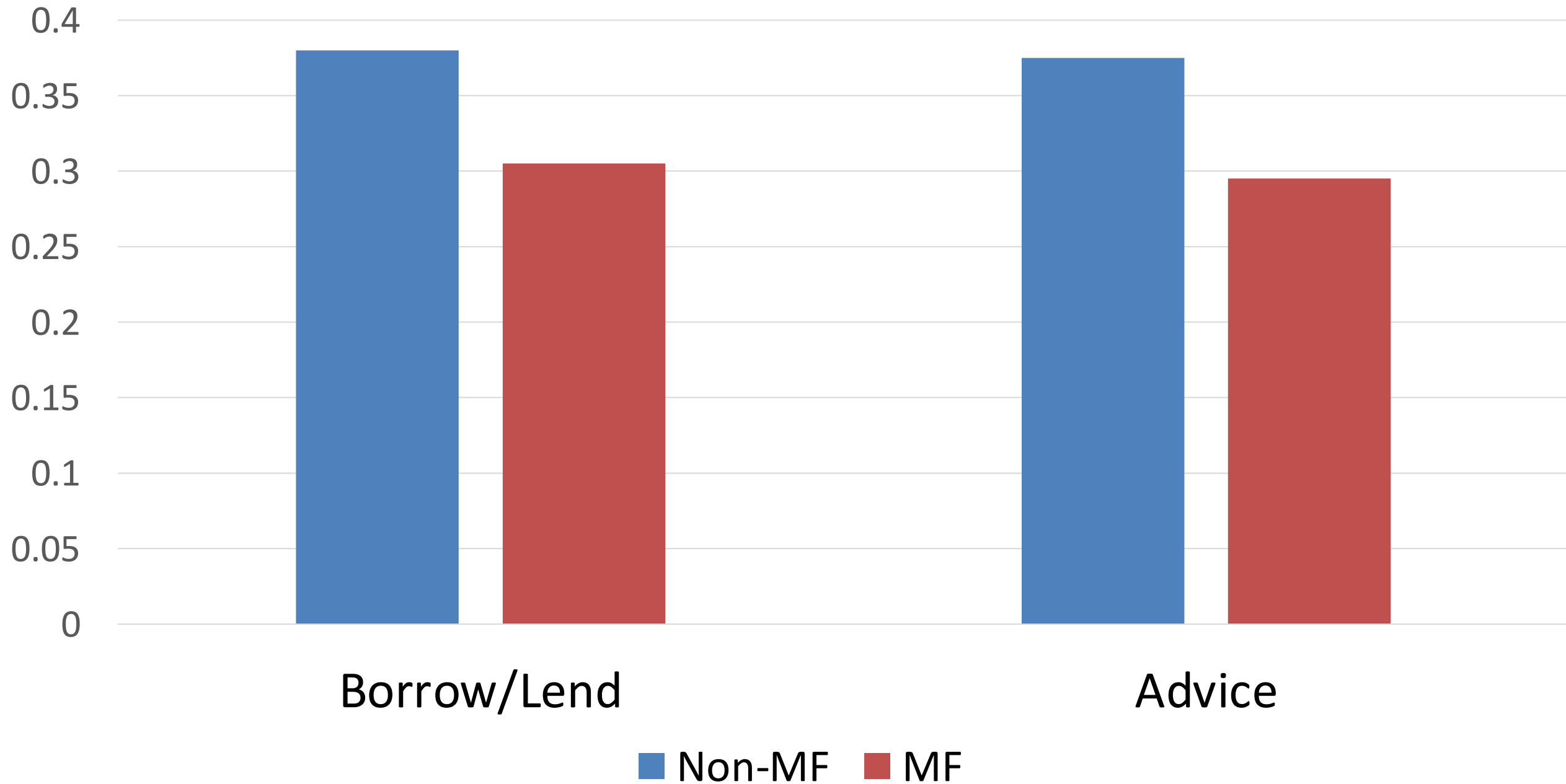
**Medic**



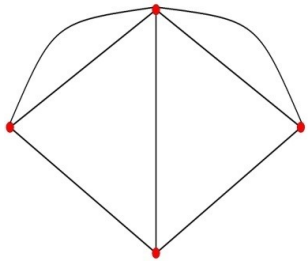
Fraction of Relationships Retained 2012-2006



Fraction of Relationships Retained 2012-2006

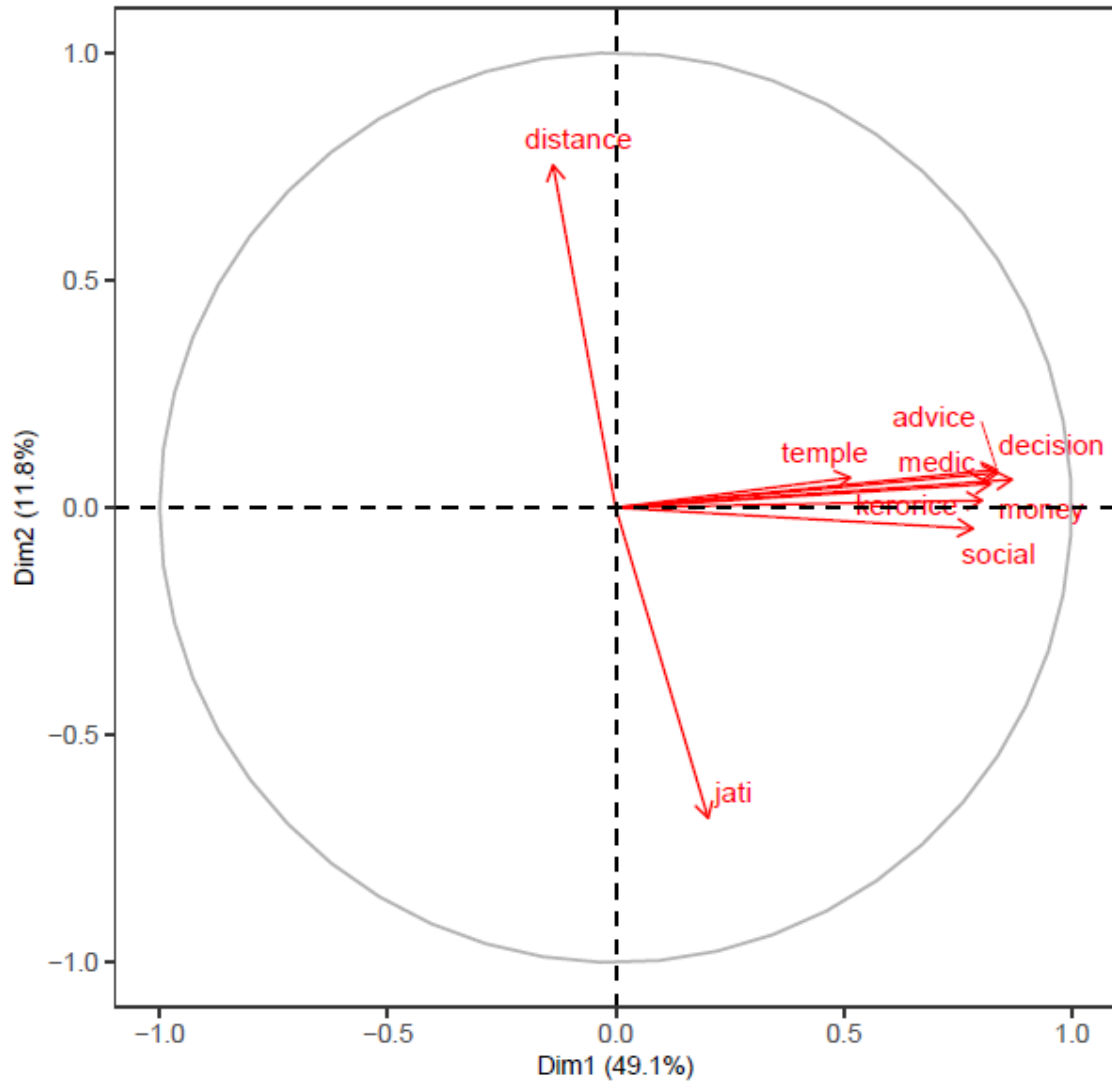


# Networks:

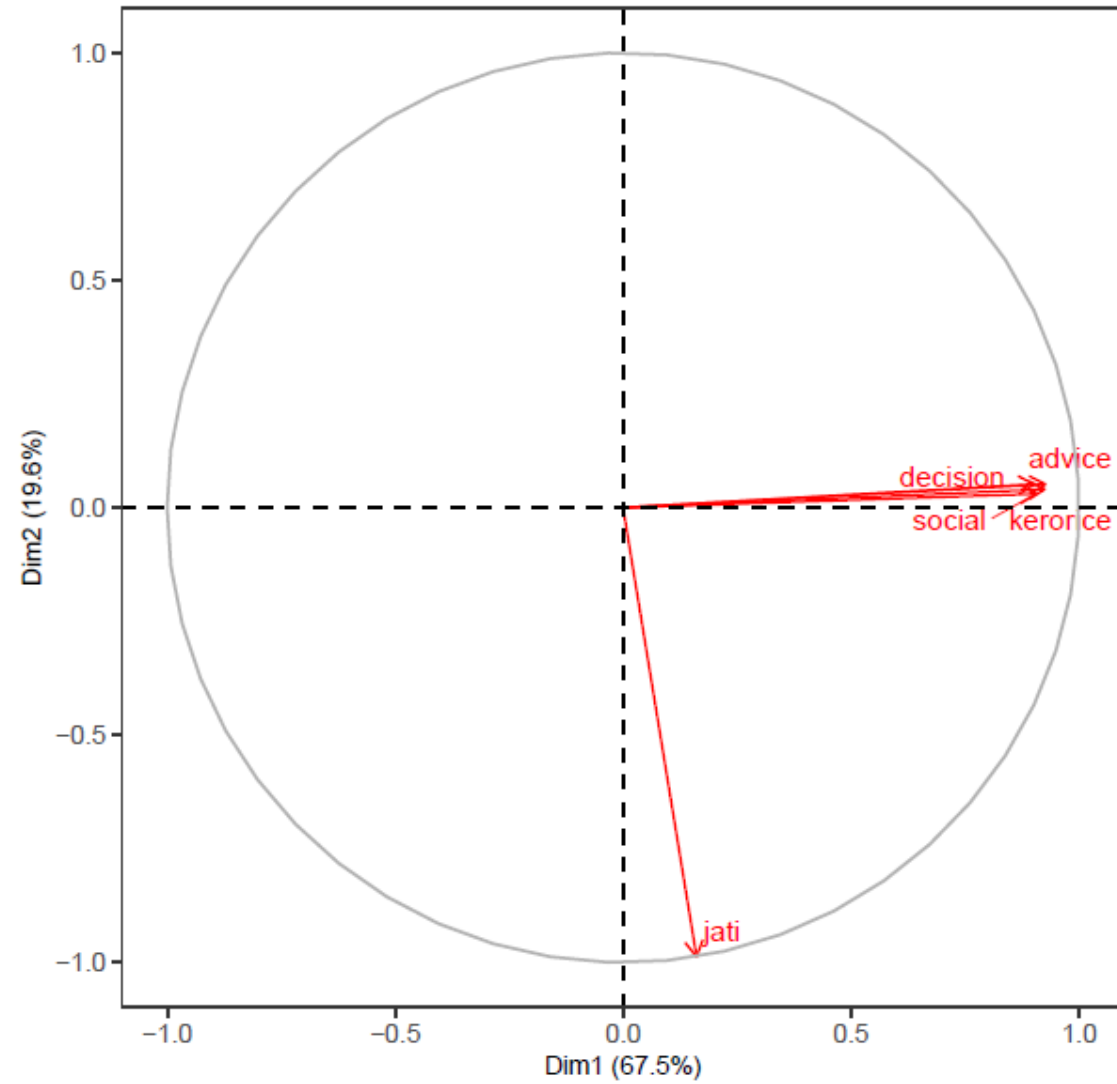


- MF (75 villages) 9 layers:
  - Kero-Rice
  - Money
  - Socialize: relatives, non, visit
  - Medical help
  - Temple
  - Advice
  - Decision Help
  - Jati
  - Geography
- RCT (68 diff villages) 5 layers:
  - Kero-Rice/Money
  - Socialize
  - Advice
  - Decision help
  - Jati

# Principal Component Analysis



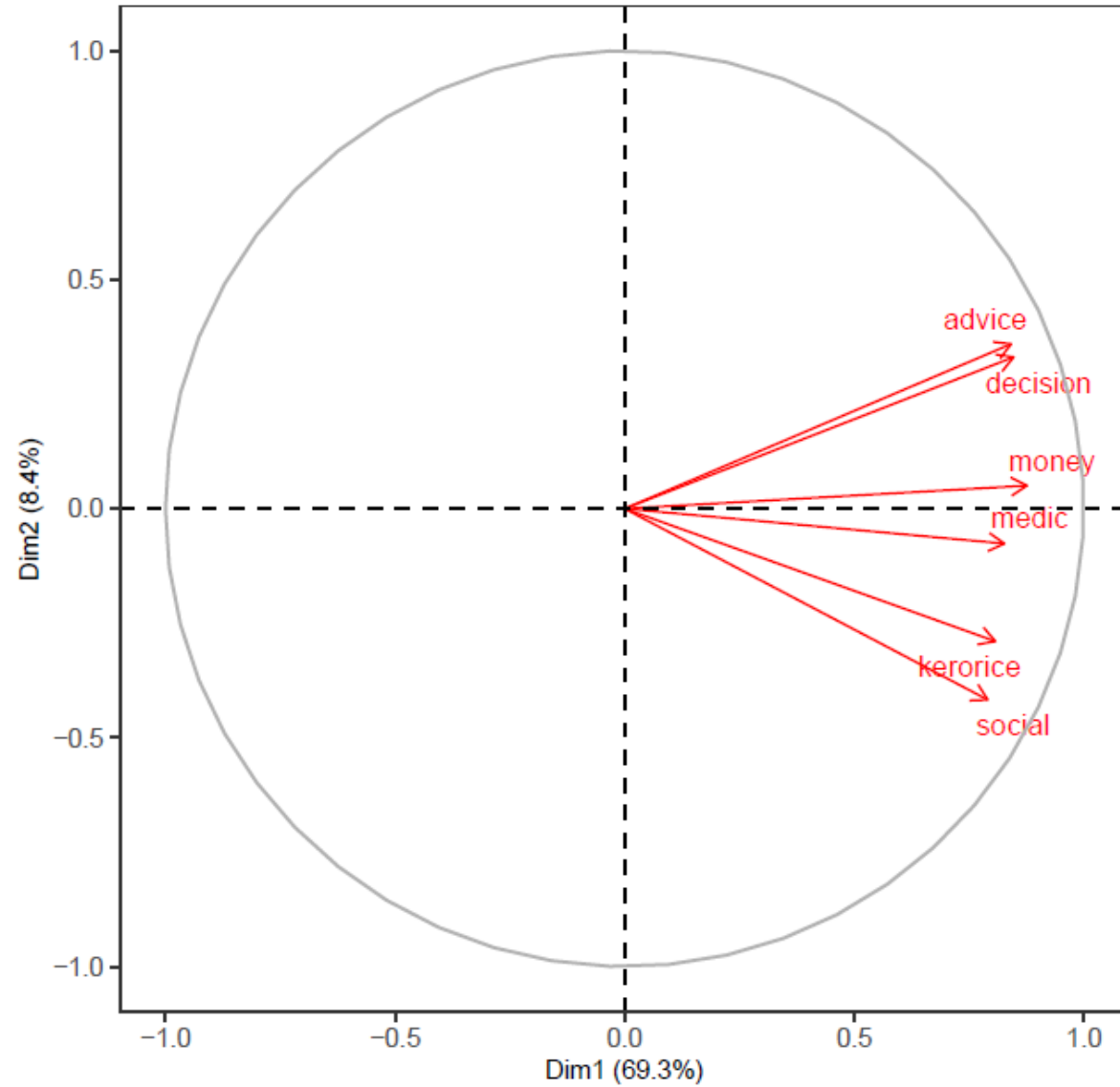
MF villages



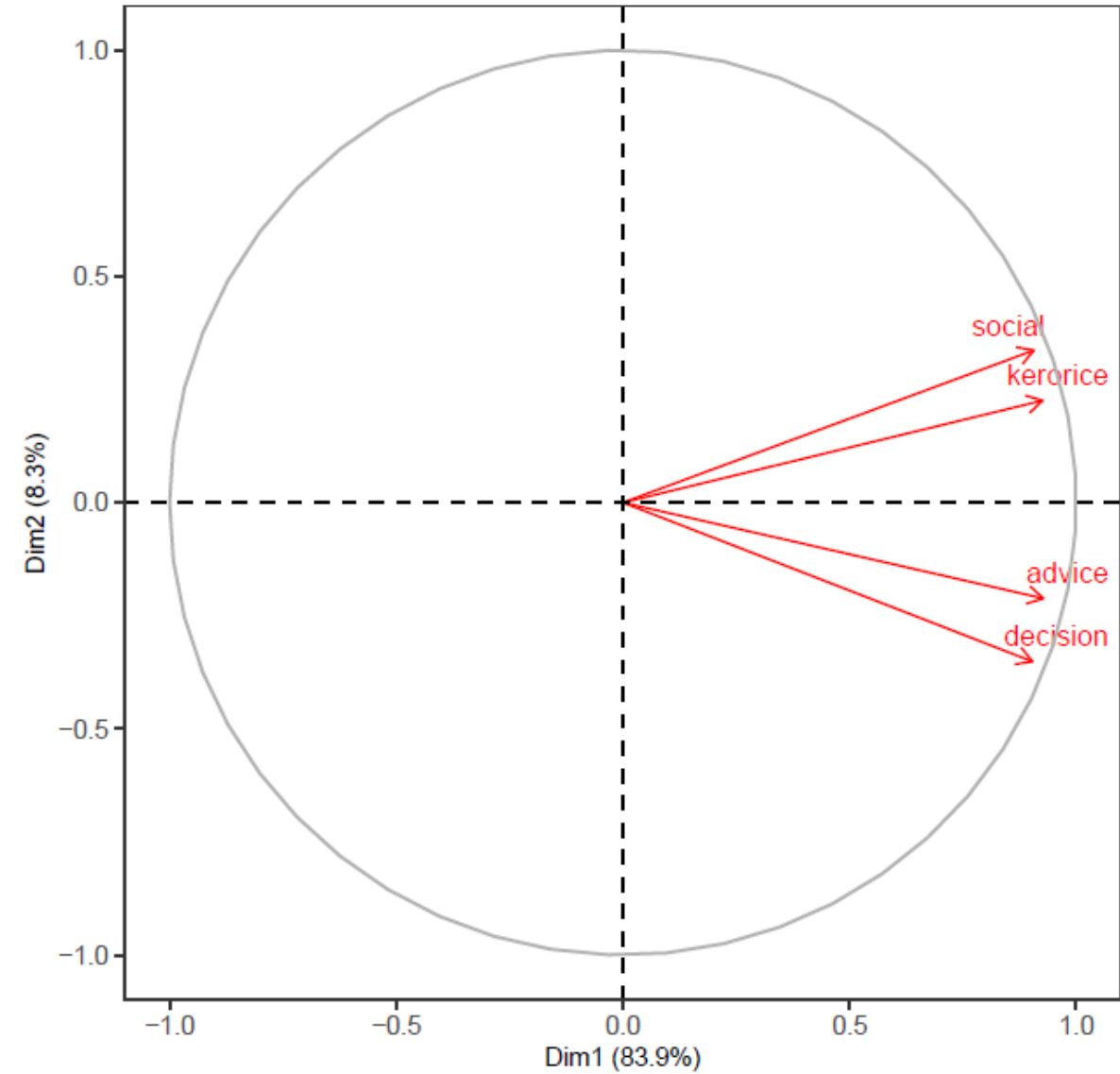
RCT villages



# Principal Component Analysis (excluding jati and geography)

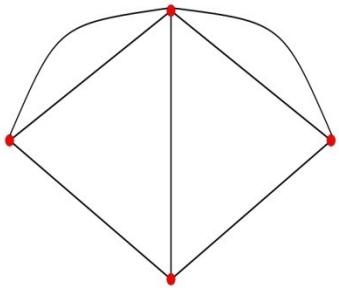


MF villages



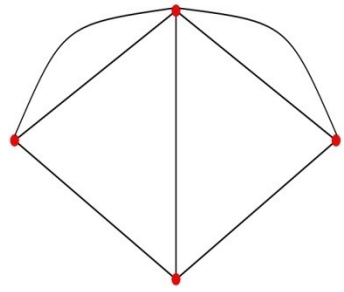
RCT villages

# Outline



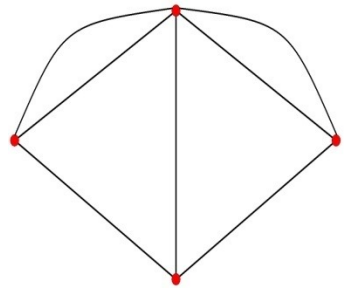
- Empirical background multiplexing
- How do multiple layers affect diffusion (in an RCT)?
- Theory on how multiplexing impacts diffusion (simple, complex)

# RCT on Diffusion



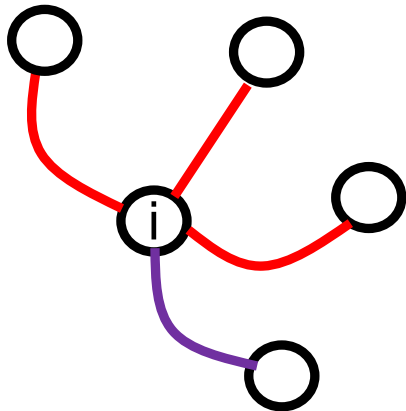
- Spread information about a chance to win a cell phone
- Randomly tell 3 to 5 people in each village and ask them spread information
- Measure diffusion of information by how many people participate in the cell-phone giveaway

# Diffusion

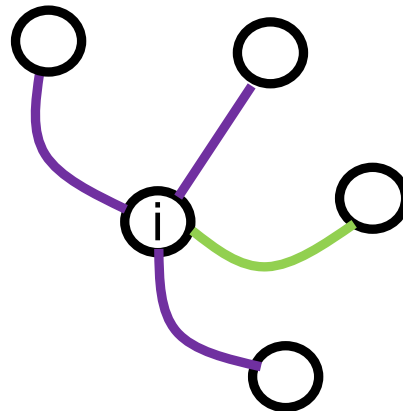


- Which network layer(s) best predict the diffusion?
  - Is it more important to have “seeds” central in advice or in kero/rice or in jati...?

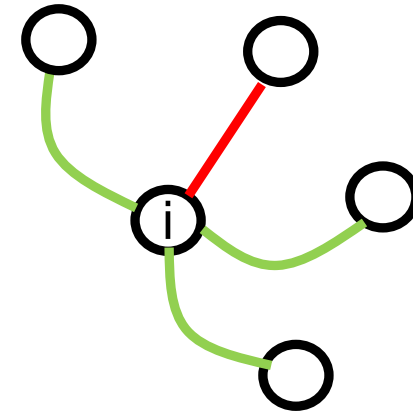
central in  
advice network



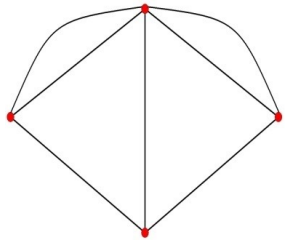
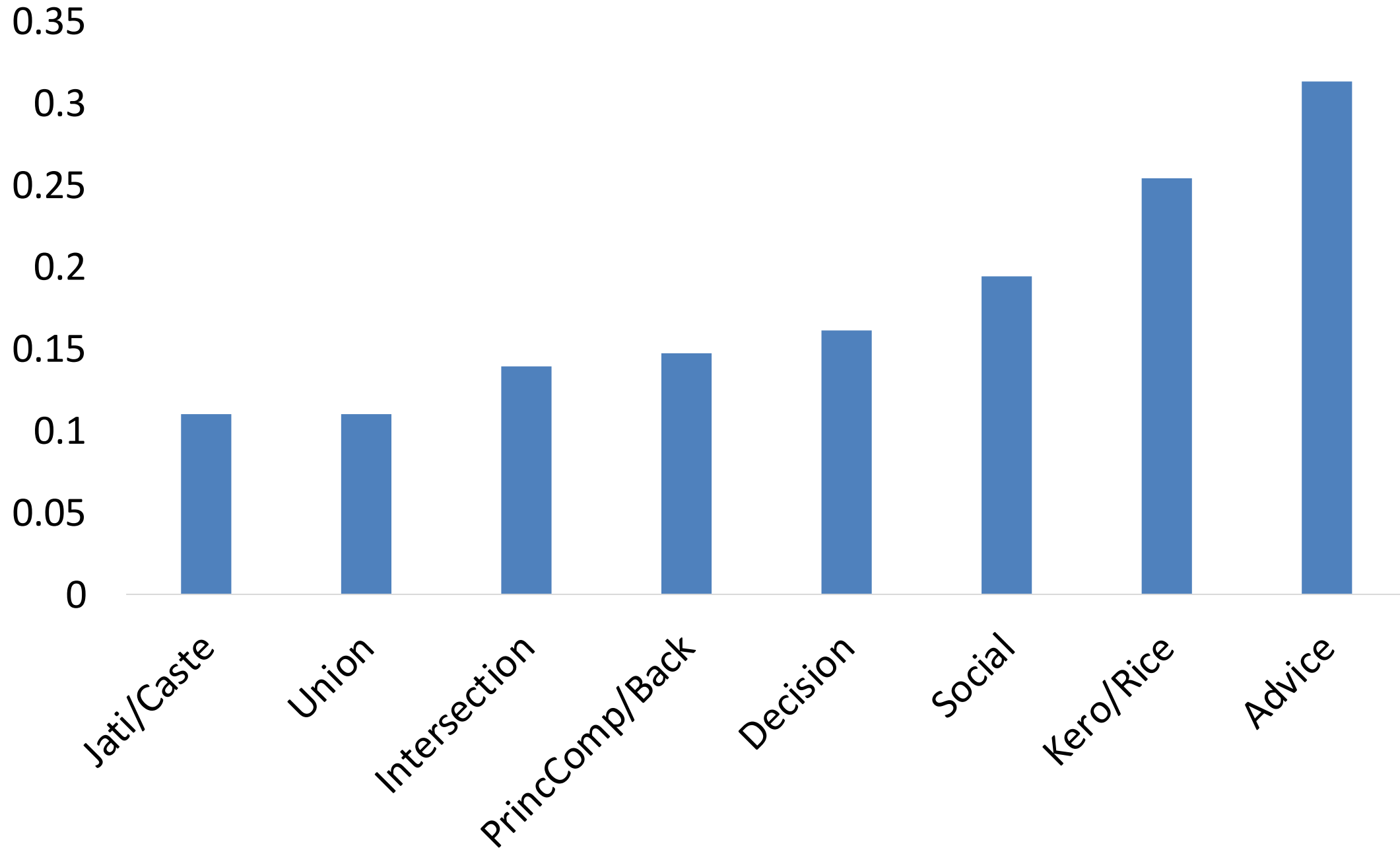
central in  
kero/rice network



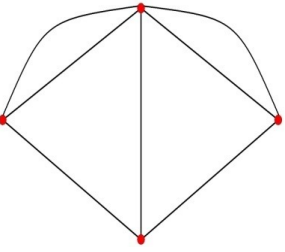
central in  
jati network



Extent of Diffusion Explained (R-sq)



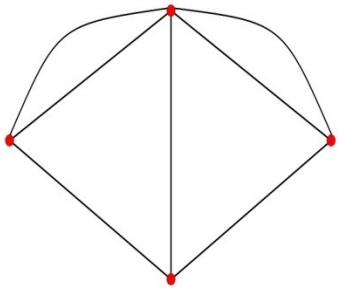
# Cumulative Predictive Power:



| layer        | df | R.sq. | F-stat | p-val |
|--------------|----|-------|--------|-------|
| advice       | 1  | 0.233 | 20.057 | 0.000 |
| intersection | 2  | 0.276 | 3.888  | 0.053 |
| kerorice     | 3  | 0.281 | 2.134  | 0.127 |
| jati         | 4  | 0.325 | 2.844  | 0.045 |
| backbone     | 5  | 0.336 | 2.415  | 0.058 |
| union        | 6  | 0.340 | 1.971  | 0.096 |
| social       | 7  | 0.342 | 1.657  | 0.147 |
| decision     | 8  | 0.344 | 1.419  | 0.215 |

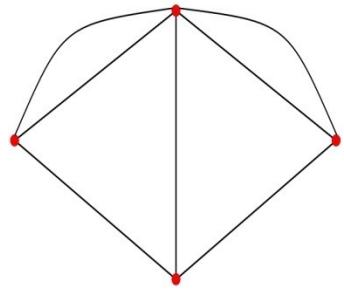
More than just advice matters:  
combinations of layers matter in predicting diffusion

# Outline



- Empirical background multiplexing
- How do multiple layers affect diffusion (in an RCT)?
- Theory on how multiplexing impacts diffusion (simple, complex)

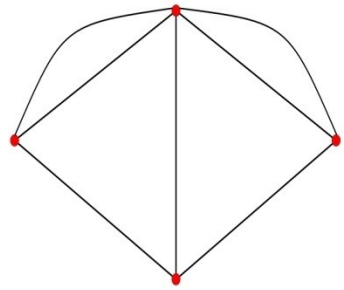
# Theory: Diffusion



- Agents either infected or susceptible
- If infected recover randomly at some rate  $\delta$  each period
- If susceptible can be infected by contact with infected neighbors

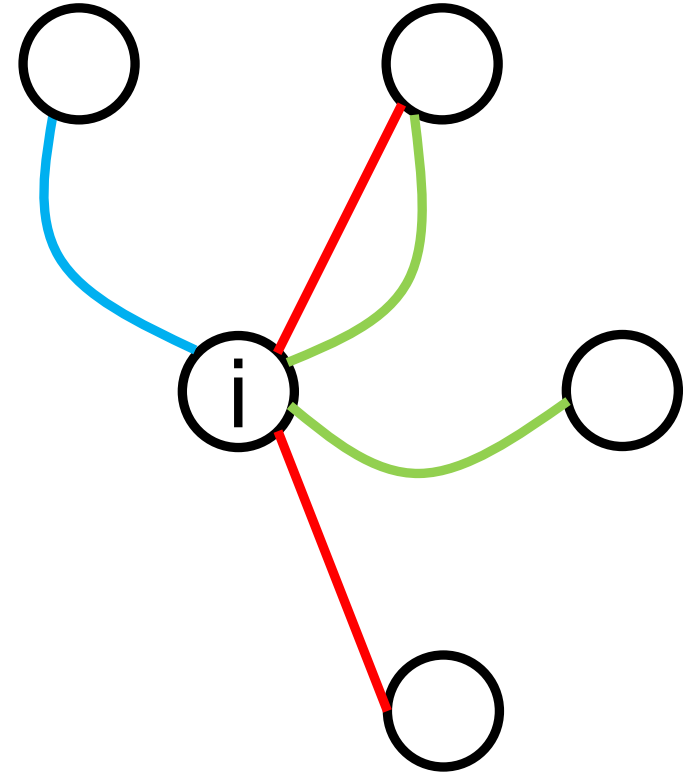
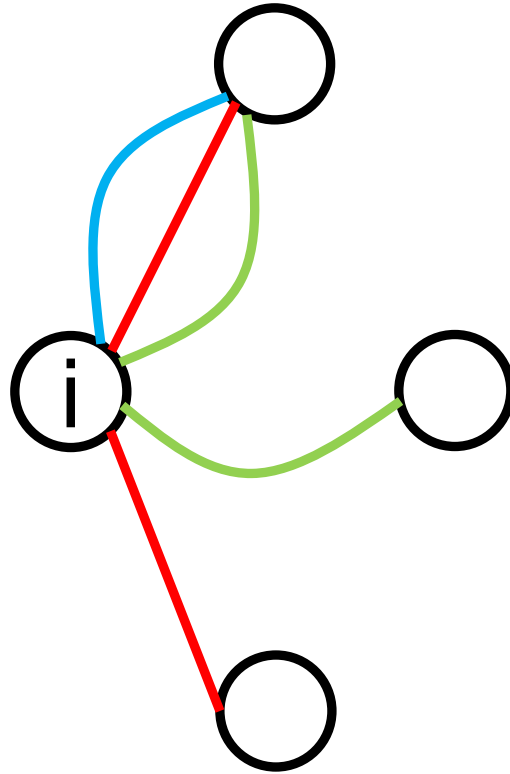
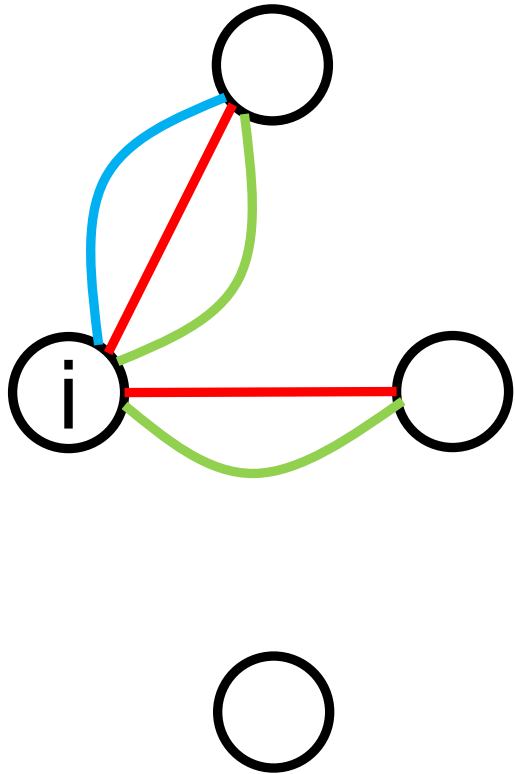
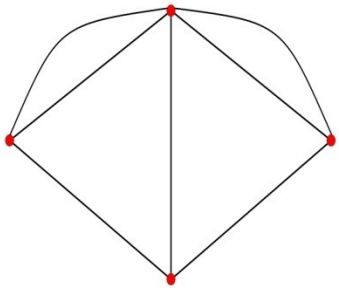


# Theory: Diffusion

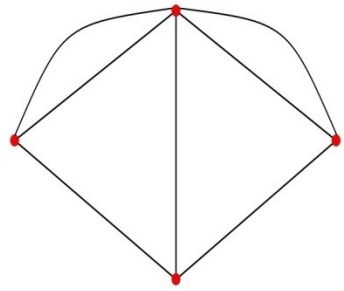


- Probability  $q_l$  that infected agent contacts susceptible on layer  $l$
- Agent becomes infected when getting at least  $\tau$  'contacts'
  - $\tau = 1$  simple contagion
  - $\tau > 1$  complex contagion
- Contacts independent across layers (not too negatively correlated)

# Less Multiplexed



# Proposition: Multiplexing Hurts Diffusion under *Simple Diffusion*

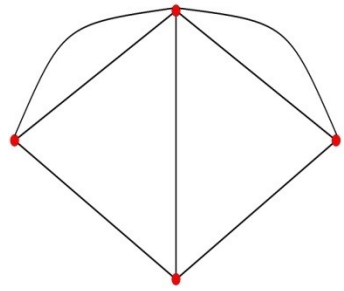


Consider agents  $i$  and  $j$ , with  $i$  *more* multiplexed than  $j$ .

If  $i$ 's and  $j$ 's neighbors are each infected with probability  $p$ , then  $i$  is *less* likely to be infected.

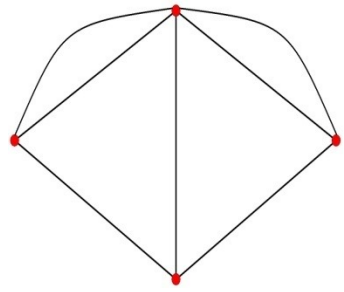
*Less multiplexing – more diffusion/contagion*

## Intuition/Proof Logic:



- $p$  chance neighbor is infected
- Infection on one multiplexed relation:  $q_A p + q_B p - q_A q_B p$
- Infection on two un-multiplexed relations:  $q_A p + q_B p - q_A q_B p^2$

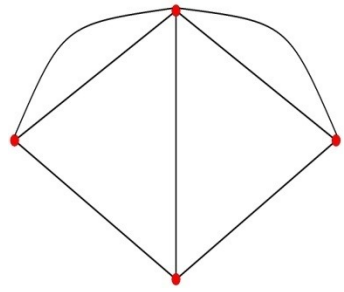
# Proposition: Multiplexing Hurts Diffusion under *Simple Contagion*



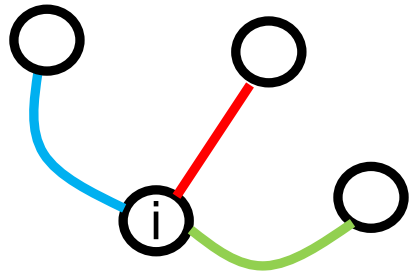
In the SIS (or SIR) model, the mean-field steady-state infection rate is **decreasing** in the multiplexing of the network.

*Less multiplexing - more diffusion/contagion*

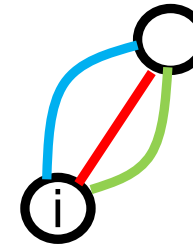
## Back to the Data



Theory: more multiplexed networks less simple diffusion/contagion



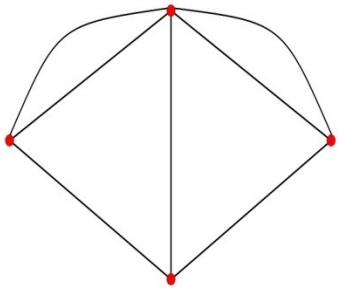
more diffusion



less diffusion

Do we see less diffusion in more multiplexed villages?

# Multiplexing Index

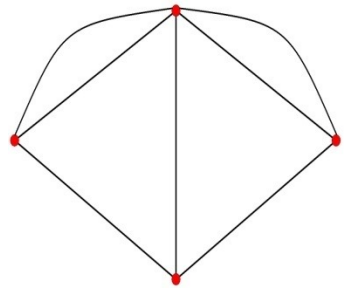


$$m_v := \frac{1}{n} \sum_i \frac{\sum_j \left( \sum_{\ell} g_{ij,v}^{\ell} / L \right)}{\sum_j 1_{\{\sum_{\ell} g_{ij,v}^{\ell} > 0\}}}$$

|                                         | Participation |
|-----------------------------------------|---------------|
| High Multiplexing x Seed Set Centrality | -.039**       |
|                                         | (.017)        |
| Seed Set Centrality                     | .052***       |
|                                         | (.016)        |
| High Multiplexing                       | -.023         |
|                                         | (.016)        |
| Observations                            | 68            |



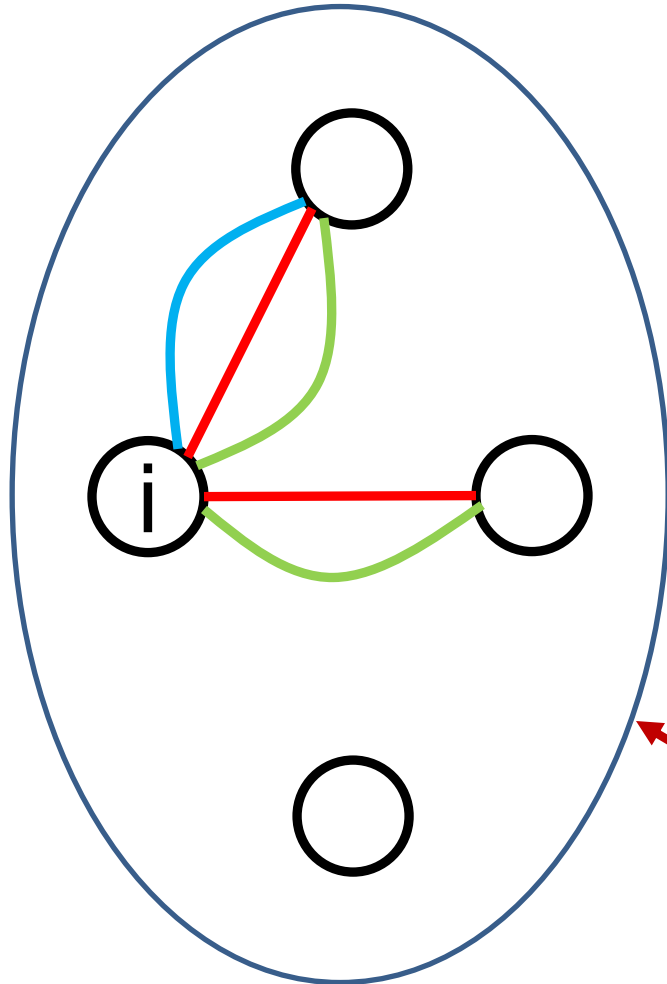
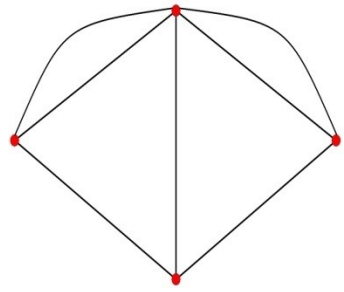
# Complex Diffusion is more Complex



No longer can order based on multiplexing

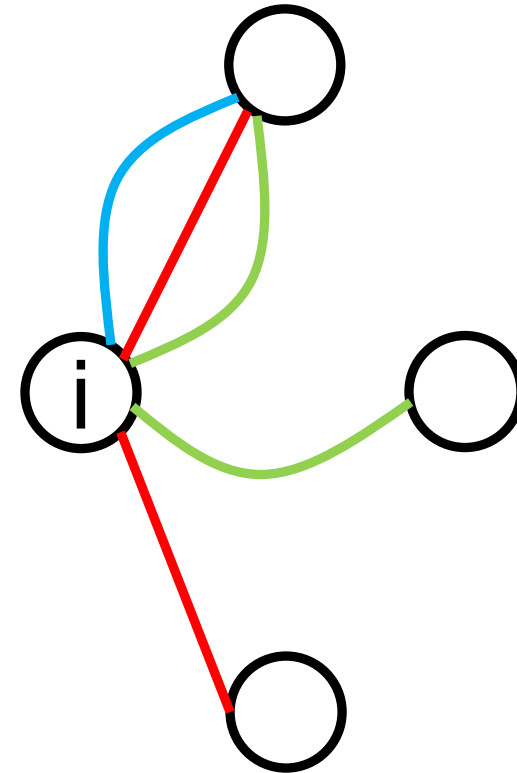
Interaction with threshold

# Complex Contagion

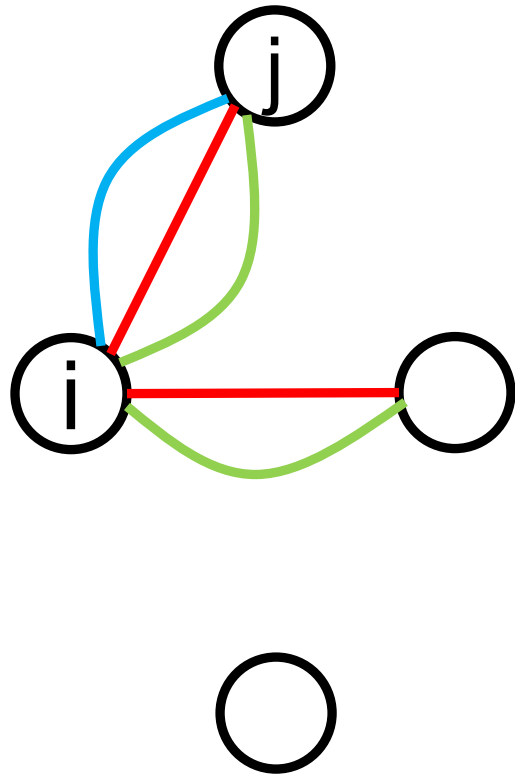
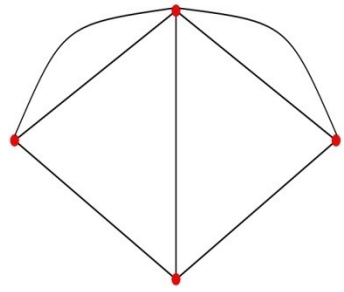


$\tau = 5$   
need all  
neighbors

Easier to infect

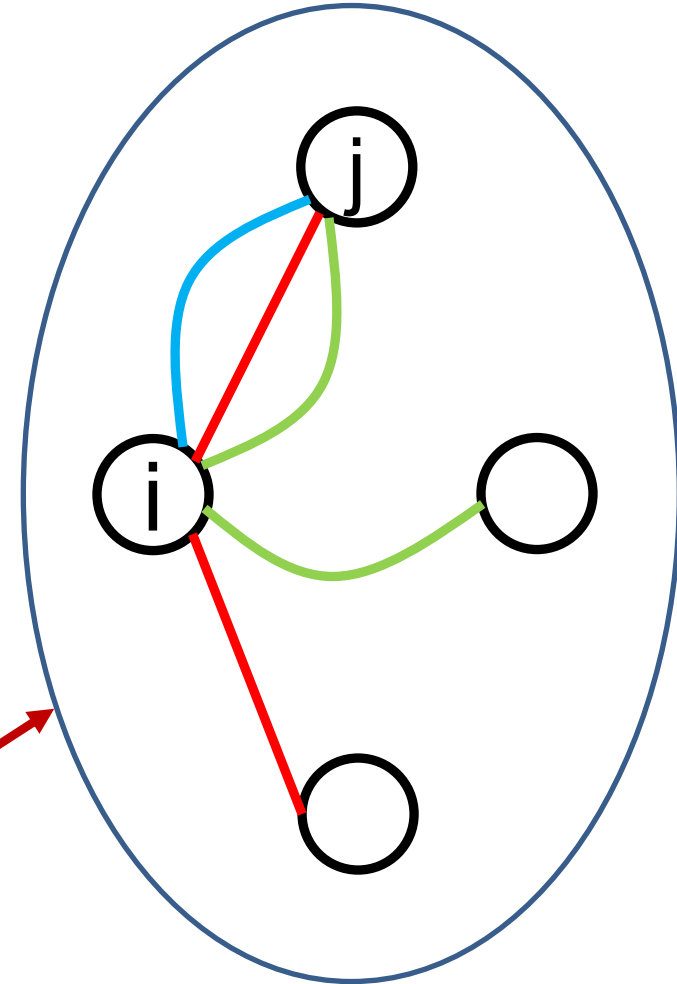


# Complex Contagion

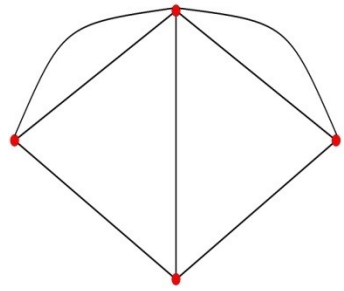


$\tau = 4$   
need  $j$  and  
one  
other

Easier to infect



# Proposition: Complex Contagion

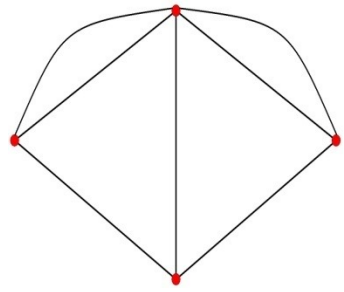


With **complex** contagion (threshold  $>1$ ), total degree summed across layers is at least  $\tau + 1$  :

There exist  $p' < p''$  (increasing in threshold) such that

- for  $p < p'$  infection probability is **increasing** in multiplexing;
- for  $p > p''$  infection probability is **decreasing** in multiplexing.

# Complex Contagion



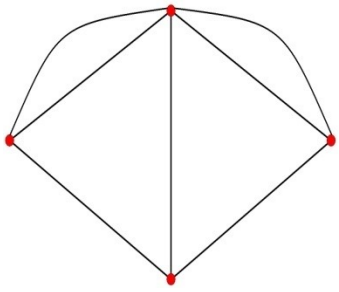
When the infection rate is low,

- Two neighbors being infected is unlikely.
- A multiplexed neighbor infected gives twice contact.

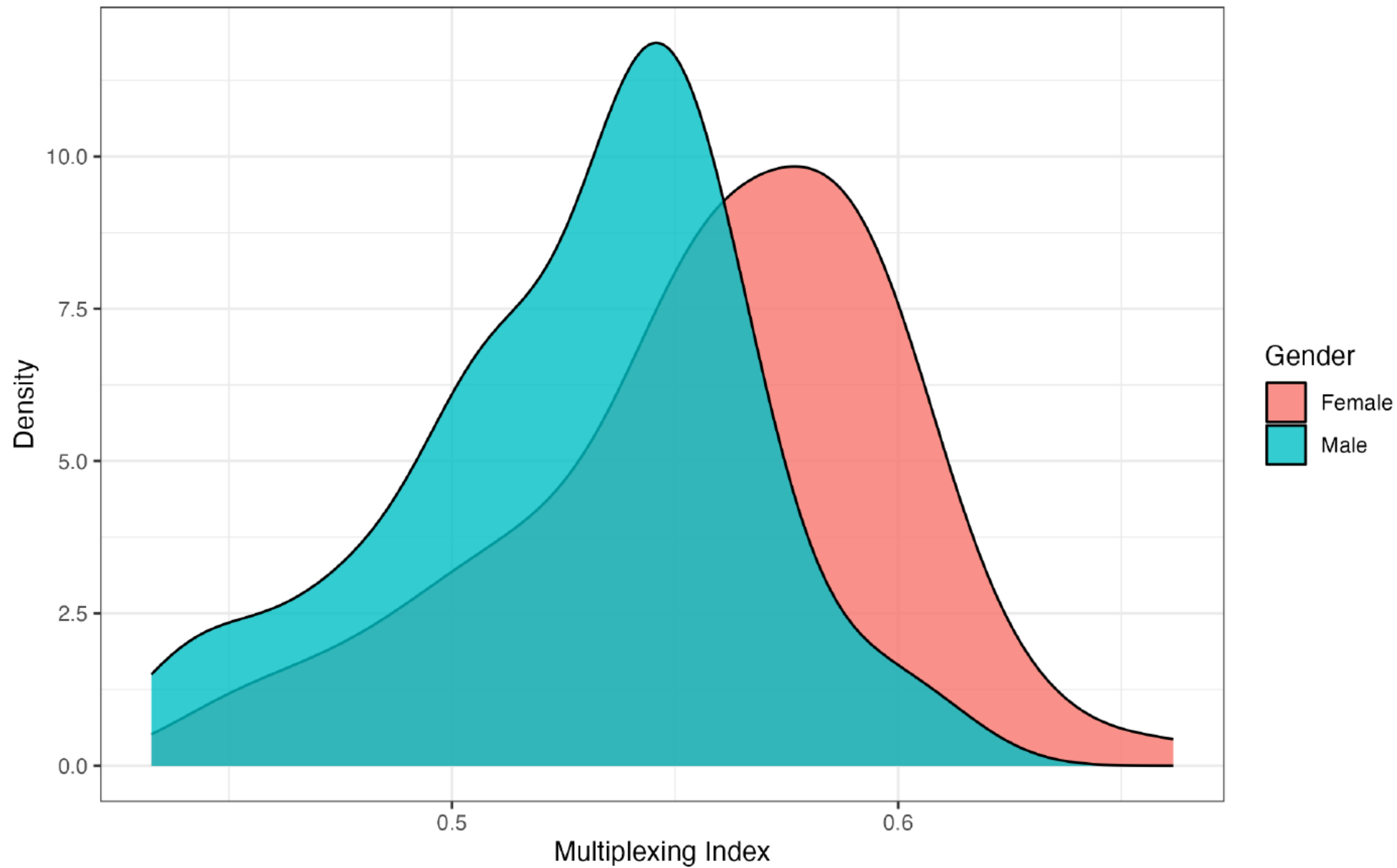
When the infection rate is higher,

- More likely that only need one contact from these two links
- Non-multiplexed more likely to get at least one contact.

# Further Observations

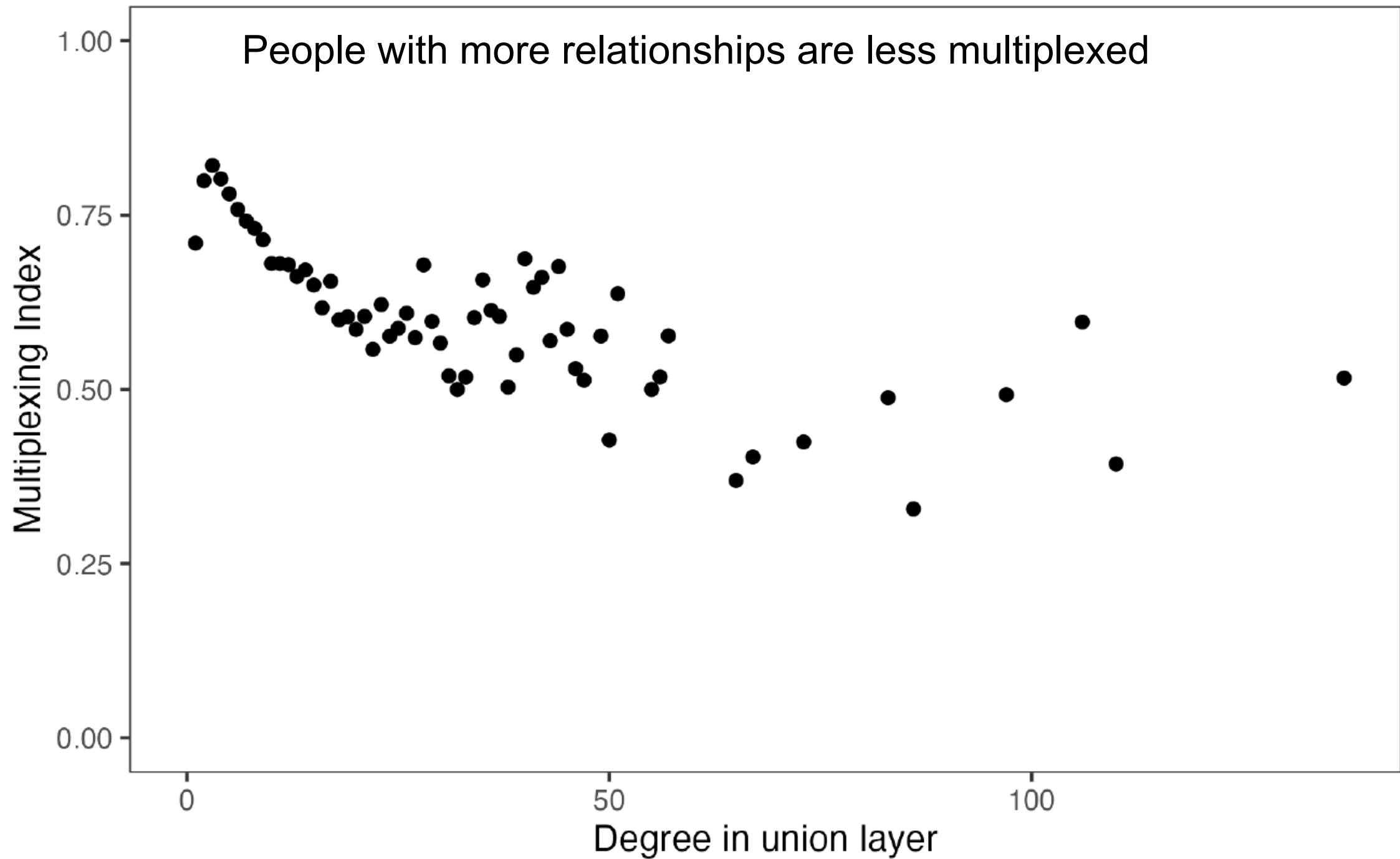


- Who is more multiplexed?

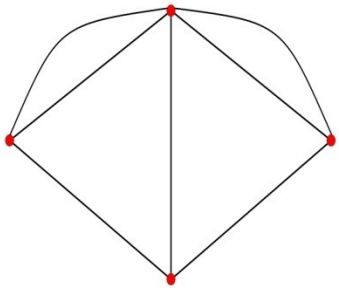


|               | High Multiplexing |
|---------------|-------------------|
| Poverty Index | .017**            |
|               | (.007)            |
|               |                   |
| Observations  | 12732             |





# Closing Thoughts



- Multiplexing matters in many contexts
  - Networks between nations (trade, migration, war)
  - Networks between companies (partnerships, lending, competition)
  - Networks among workers (communication, direction, collaboration)
  - Networks among students (friends, study partners, roommates)
  - ....
- Need for more theory/empirics of multiplexing and behaviors, network formation, methods...