Queueing Games: The role of customers' heterogeneity

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51st LNMB Conference on the Mathematics of Operations Research
January 2026, Soesterberg

Part I:

Queueing Games with heterogeneous customers: A survey

Framework I

- Majority of studies about service systems with strategic customers assume **homogeneous** customers regarding economic parameters:
  - * Service value: R for all customers.
  - * Waiting cost per time unit: C for all customers.

- But, in reality, customers are **heterogeneous**:
  - * R and C are random variables refering to the population of customers.

Framework II

- **Common approach** when studying a service system with a heterogeneous population of customers:
  - Compute/estimate the mean values of the random variables R and C.
  - Use the results for the homogeneous counterpart with R and C being these mean values.

Main questions

- What is the equilibrium customer behavior for join-or-balk in a system with heterogeneous customers?
- How does customer heterogeneity affect a system?
  - * the equilibrium strategies?
  - * the equilibrium customers' surplus?
  - * the equilibrium administrator's profit?
  - * the equilibrium social welfare?
- Is the common approach reliable?
  - * How much does the administrator lose when he ignores heterogeneity in pricing service?

The model: Operational characteristics

- M/M/1 queue:
  - * Poisson arrival process at rate λ,
  - * Exp(μ) service times,
  - * 1 server,
  - * Unlimited waiting space.

The model: A general cost-reward structure

- Heterogeneous customers
  - * in service value,
  - * in waiting cost per time unit.

- Arriving customers parameterized by a pair (r, c).

- An (r, c)-customer has
  - * service value: r,
  - * waiting cost per time unit: c.

- The parameter pairs (r, c) that correspond to different customers are realizations of non-negative i.i.d. r.v. (R, C) with some known distribution.

References

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Part II:

Join-or-balk dilemma in the case of totally positive correlation between service value and waiting cost rate

The model: operational, economic characterists

- M/M/1 queue with arrival rate λ and service rate μ.
- Heterogeneous customers parameterized by their delay-sensitivity, c. A c-customer has
  - * waiting cost per time unit: c
  - * service value: $r(c) = v + dc$, where $v, d \geq 0$ are population constants.

  totally positive correlation
  between service value and waiting cost rate.
- I.i.d. delay-sensitivity parameters for the customers distributed as a r.v. C on $[c_L, c_U]$, with distr. $H(c)$.
- p: Entrance fee imposed by the service provider.

The model: decision framework

- Information: system parameters (unobserv. system).
- Customers:
  * Decision: join or balk.
  * Objective: utility maximization.
- Administrator:
  * Decision: entrance fee.
  * Objective: revenue or social welfare maximization.

The queueing games

Fixed-fee case:

Exogenously given entrance fee
(the service provider does not set the price)
→ game among customers.

Pricing case:

Revenue-maximizing entrance fee set by service provider
→ two-stage game of customers and service provider

- 1st stage: the service provider sets the fee
- 2nd stage: game among customers (given the fee).

Utility functions

- Customers:
  Utility of a c-customer who decides to enter, when the population follows strategy $\mathbf{s}$:

$$\mathcal{U}(c|\mathbf{s}) = r - p - cW(\mathbf{s}),$$

  where $W(\mathbf{s})$ is the expected sojourn time under $\mathbf{s}$.

- Service provider: Utility (revenue) of the service provider when a fraction $q(\mathbf{s})$ of customers enter under population strategy $\mathbf{s}$:

$$\mathcal{P}(s) = \lambda q(\mathbf{s})p.$$

Economic interpretations

- Service value: $r(c) = v + dc$
  - * The more a customer values service the more impacient she is to receive it.
  - * v is a base service value that does not take into account the sensitivity to wait and is common for all customers.
  - * d quantifies how strongly a customer's impatience to receive service reflects on the value of the service.

- Utility: $\mathcal{U}(c|\mathbf{s}) = v - p + c(d - W(\mathbf{s}))$
  - * $v - p$ is the net value of the service without considerations about sensitivity to wait.
  - * d is a critical value for the waiting time.
    $W(\mathbf{s}) > d \Rightarrow$ Utility $\downarrow$ w.r.t. the sensitivity to wait.
    $W(\mathbf{s}) < d \Rightarrow$ Utility $\uparrow$ w.r.t. the sensitivity to wait.

Best responses

- Assume that customers follow strategy $\mathbf{s}$

$$\mathbf{s} : [c_L, c_U] \to [0, 1]$$

$\mathbf{s}(c)$: joining probability of a c-type customer
$q(\mathbf{s}) = \int_{c_L}^{c_U} s(c)dH(c)$
: fraction of joining customers under strategy $\mathbf{s}$

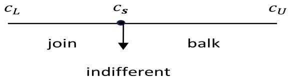
- Consider a c-type arriving customer.

$$\begin{aligned} \mathcal{U}(c|\mathbf{s}) &= \overbrace{v + dc}^{\text{service value}} - \overbrace{p}^{\text{fee}} - \overbrace{c\frac{1}{\mu - \lambda q(\mathbf{s})}}^{\text{mean waiting cost}} \\ &= v - p + c\left(d - \frac{1}{\mu - \lambda q(\mathbf{s})}\right). \end{aligned}$$

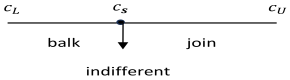
Best responses (cont.)

Let $c_{\mathbf{s}} = \frac{p-v}{d-\frac{1}{\mu-\lambda q(\mathbf{s})}}$ be the solution of $\mathcal{U}(c|\mathbf{s}) = 0$.

- $d - \frac{1}{\mu-\lambda q(\mathbf{s})} < 0 \Rightarrow \mathcal{U}(c|\mathbf{s}) \downarrow c \Rightarrow$ BR=threshold strat.



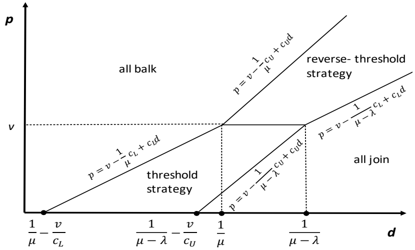
- $d - \frac{1}{\mu-\lambda q(\mathbf{s})} > 0 \Rightarrow \mathcal{U}(c|\mathbf{s}) \uparrow c \Rightarrow$ BR= reverse-thres. strat.



- $d - \frac{1}{\mu-\lambda q(s)} = 0 \Rightarrow \mathcal{U}(c|\mathbf{s}) = v - p \Rightarrow$

$$\text{BR} = \begin{cases} \text{all join} & \text{if } p < v, \\ \text{all balk} & \text{if } p > v, \\ \text{any strategy} & \text{if } p = v. \end{cases}$$

Equilibrium Strategies w.r.t. $(p.d)$: Results



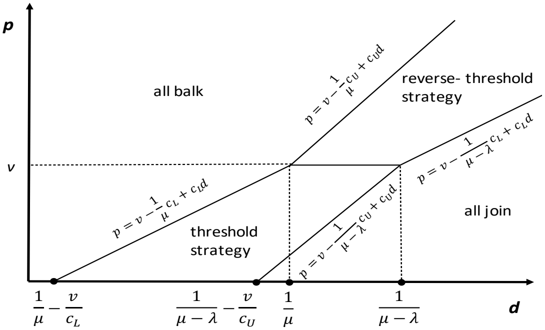
- When (d, p) lies in the threshold strategy area of the quarter plane, the threshold c_0 is the unique solution of

$$\frac{p-v}{c_0} + \frac{1}{\mu - \lambda H(c_0)} = d$$

- When (d, p) lies in the reverse-threshold strategy area of the quarter plane, the reverse-threshold c_0 is the unique solution of

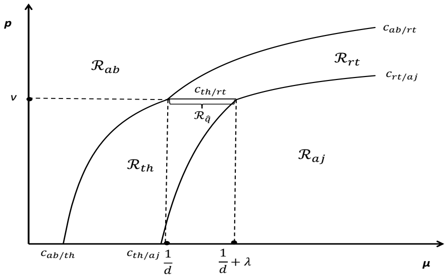
$$\frac{p-v}{c_0} + \frac{1}{\mu - \lambda(1 - H(c_0))} = d$$

Equilibrium Strategies: Interpretations



- An equilibrium strategy always exists and is almost unique.
- For unfavorable system characteristics (high prices, low critical waiting times) all customers balk.
- For favorable system characteristics (low prices, high critical waiting times) all customers join.

Equilibrium Strategies w.r.t. (p, μ)



- For high prices:
  - * all customers balk, if the service is slow,
  - * all customers join, if the service is fast,
  - * only customers with high c join if moderate service.
- For low prices:
  - * all customers balk, if the service is slow,
  - * all customers join, if the service is fast,
  - * only customers with low c join if moderate service.

Part III:
Comparing systems with different degrees of heterogeneity

Effect of Heterogeneity: Framework I

- We want to explore the effect of heterogeneity on
  - * the equilibrium strategy
  - * the customer surplus per time unit, $\mathcal{CS}$
  - * the service provider's profit, $\mathcal{P}$

- To this end, we consider two systems with identical operational and economic parameters: λ, μ, p, v, d which differ only in the distribution of C.

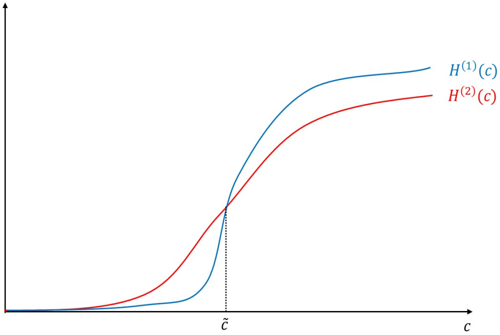
- Two populations:
  - * Popul. 1: $C^{(1)}$, with cdf $H^{(1)}(c)$, $E[C^{(1)}]$, $Var[C^{(1)}]$.
  - * Popul. 2: $C^{(2)}$, with cdf $H^{(2)}(c)$, $E[C^{(2)}]$, $Var[C^{(2)}]$.
  - * Population 2 more heterogeneous than population 1.

Effect of Heterogeneity: Framework II

- How to model that population 2 is more heterogeneous than population 1?
- Option 1: equality of means + order of variances $E[C^{(1)}] = E[C^{(2)}]$ and $Var[C^{(1)}] \leq Var[C^{(2)}]$.
- Option 2: convex order: $C^{(1)} \leq_{cx} C^{(2)} \Leftrightarrow E[\phi(C^{(1)})] \leq E[\phi(C^{(2)}], \forall \phi$ convex.
- Option 3: equality of means + less dangerous order: $E[C^{(1)}] = E[C^{(2)}]$ and

$$\exists \tilde{c}: \quad \begin{array}{ll} H^{(1)}(c) \leq H^{(2)}(c), & \text{for } c \leq \tilde{c} \\ H^{(2)}(c) \leq H^{(1)}(c), & \text{for } c \geq \tilde{c}. \end{array}$$

Effect of Heterogeneity: Framework III



- Option 3: equality of means + less dangerous order:
  $E[C^{(1)}] = E[C^{(2)}]$
  and

$$\exists \tilde{c}: \quad \begin{array}{ll} H^{(1)}(c) \leq H^{(2)}(c), & \text{for } c \leq \tilde{c} \\ H^{(2)}(c) \leq H^{(1)}(c), & \text{for } c \geq \tilde{c}. \end{array}$$

Effect of Heterogeneity: Framework IV

- Option 4: location-scale transformation that preserves mean value and increases variance:

$$C^{(2)} = \sigma(C^{(1)} - E[C^{(1)}]) + E[C^{(1)}], \quad \sigma > 1.$$

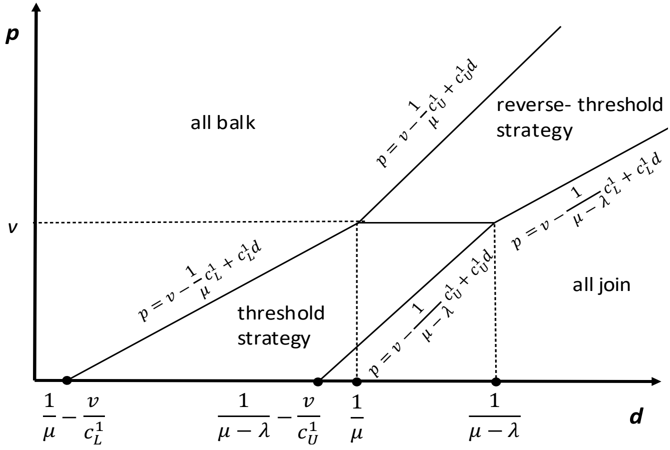
- Relationship:
  Option 4 $\Rightarrow$ Option 3 $\Rightarrow$ Option 2 $\Rightarrow$ Option 1.

- We will use option 3 and write $C^{(1)} \leq_{heter} C^{(2)}$.

- Example: Option 4 $\Rightarrow$ Option 3.
  If $C^{(1)} \sim U([\mu - \alpha, \mu + \alpha])$, $C^{(2)} \sim U([\mu - \beta, \mu + \beta])$ with $\beta > \alpha$, then $C^{(1)} \leq_{heter} C^{(2)}$.

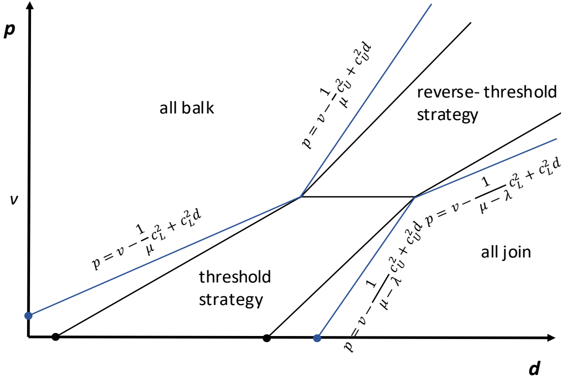
Effect of Heterogeneity: Equil. strategies I

Equilibrium strategies for population 1 (less heterogeneous)



Effect of Heterogeneity: Equil. strategies II

Equilibrium strategies for population 2 (more heterogeneous)



- blue: more heterogeneous
- black: less heterogeneous

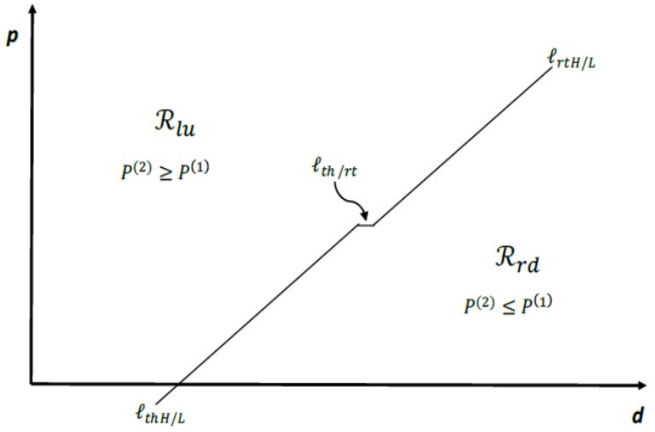
Effect of Heterogeneity: Equil. strategies II

- More heterogeneity implies that the 'all-balk' and 'all-join' areas of the (d, p)-plane shrink:

  For more values of d, p the customers of the more heterogeneous population adopt threshold or reverse-threshold strategies.

Effect of Heterogeneity: Profit

Comparisons



Effect of Heterogeneity: Profits

- More heterogeneity may have positive or negative effect on the service provider's profit.

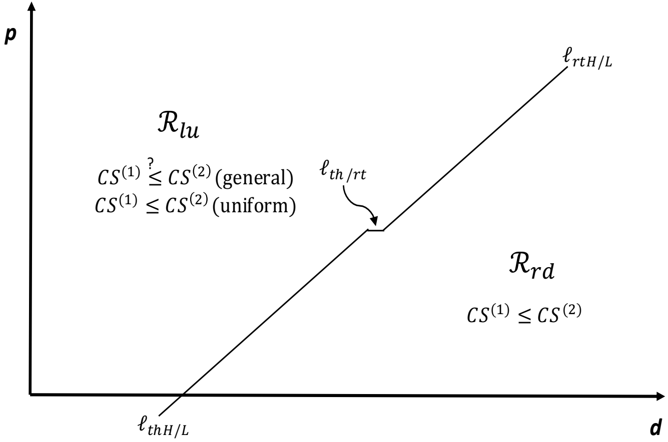
- There is a line that divides the (d, p)- quarter plane in a left-upper part ($\mathcal{R}_{lu}$) and a right-down part ($\mathcal{R}_{rd}$):

  $(d, p) \in \mathcal{R}_{lu} \Rightarrow P^{(2)} \geq P^{(1)}$:
  Under unfavorable economic parameters (high prices, low service values),
  heterogeneity benefits the service provider.

  $(d, p) \in \mathcal{R}_{rd} \Rightarrow P^{(2)} \leq P^{(1)}$:
  Under favorable economic parameters (low prices, high service values),
  heterogeneity harms the service provider.

Effect of Heterogeneity: Customer Surplus

Comparisons



Effect of Heterogeneity: Customer Surplus

- More heterogeneity has always a positive effect on customers' surplus when the delay sensitivity is uniformly distributed:

$$CS^{(2)} \geq CS^{(1)}.$$

- The inequality has been proved also for general distributions when (p, d) are below the line that separates the profit cases.

- Numerical experiments show that the inequality is valid for any distribution of the delay sensitivity and for all values of (p, d).

Part IV:

PoIH: The price of ignoring heterogeneity

Pricing

- A monopolist has to solve the problem

$$\max_{p \geq 0} \mathcal{P}(p).$$

- To induce a c_0-reverse-threshold strategy, he has to set

$$p = v + \left(d - \frac{1}{\mu - \lambda(1 - H(c_0))}\right) c_0$$

so that a c_0-customer becomes indifferent between joining and balking.

- His profit will be

$$\begin{aligned} \mathcal{P}(p) &= \lambda(1 - H(c_0))p \\ &= \lambda(1 - H(c_0)) \left[v + \left(d - \frac{1}{\mu - \lambda(1 - H(c_0))}\right) c_0 \right] \end{aligned}$$

Pricing

- Similarly for threshold strategies etc.
- Pricing is a difficult problem:
  $\mathcal{P}(p)$ is not a convex or unimodal function of p. It may have multiple local maxima.
  The global best price p^* is hard to characterize.

Naive pricing

- Consider the homogeneous alternative: Suppose that all customers have unit waiting cost $\mu_c = E[C]$ and service value $v + d\mu_c$.

- The profit maximizing price for the homogeneous problem is found in closed form:

$$p^{*hom} = \begin{cases} v + d\mu_c - \frac{\mu_c}{\mu-\lambda} & \text{if } \lambda < \mu - \sqrt{\frac{\mu_c\mu}{v+d\mu_c}}, \\ v + d\mu_c - \sqrt{\frac{(v+d\mu_c)\mu_c}{\mu}} & \text{if } \lambda \geq \mu - \sqrt{\frac{\mu_c\mu}{v+d\mu_c}}. \end{cases}$$

Price of Ignoring Heterogeneity (PoIH)

- $P(p)$: the profit function for the heterogeneous pricing problem.
- p^*: the optimal price for the heterogeneous pricing problem: $P(p^*) = \max_{p \geq 0} P(p)$.
- p^{*hom}: the optimal price for the homogeneous pricing problem.
- Price of Ignoring Heterogeneity:

$$PoIH = \frac{P(p^*)}{P(p^{*hom})}$$

A measure of how much larger is the profit if the monopolist takes into account heterogeneity in comparison to the naive approach.

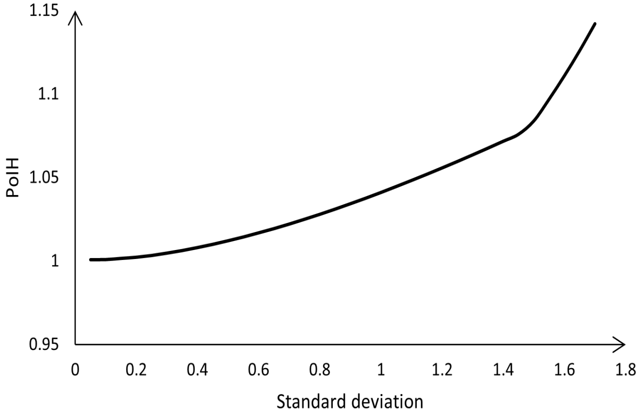
Numerical Results

- M/M/1 queue with $\lambda = 1$, $\mu \in \{0.6, 0.55, 0.4\}$.
- Basic economic parameters: $v = 1$, $d = 1$.
- Unit waiting cost C follows a uniform distribution with mean $\mu_c = 3$ and standard deviation $\sigma_c \in [0, \sqrt{3}]$.
- By increasing σ_c, we increase the heterogeneity of the population (in the sense of $\leq_{heter}$ order).

Effect of Heterogeneity on PoIH - I

- $\mu = 0.6$

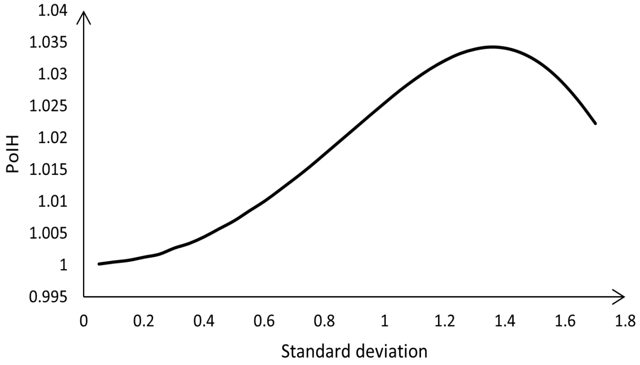
PoIH vs. heterogeneity



Effect of Heterogeneity on PoIH - II

- $\mu = 0.55$

PoIH vs. heterogeneity



Effect of Heterogeneity on PoIH - III

- $\mu = 0.4$
- $P(p^{*hom}) = 0.$
- For low values of σ_c (low heterogeneity), $P(p^*) = 0$ as well.
- For high values of σ_c (high heterogeneity) $P(p^*) > 0$ so PoIH is infinite.

Takeaway conclusions

- For exogenously given prices, in systems with unfavorable characteristics, heterogeneity benefits the service provider.
- For exogenously given prices, in systems with favorable characteristics, heterogeneity harms the service provider.
- For exogenously given prices, heterogeneity benefits the customers.
- For the pricing problem, ignoring heterogeneity may lead to significant losses in revenue. More research is needed.

Thank you!