Queueing Games: The role of information

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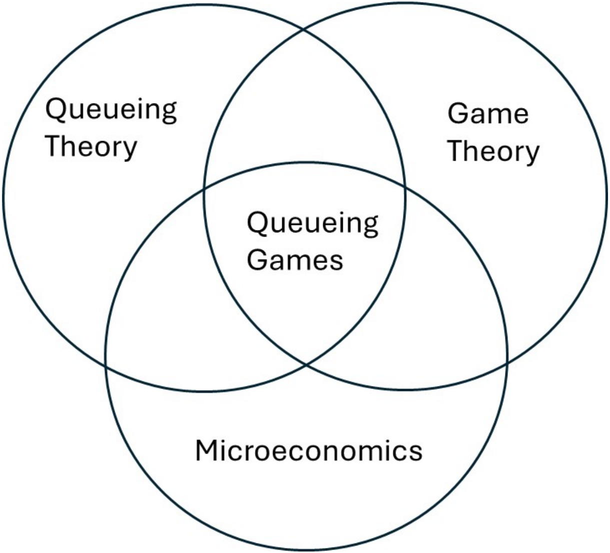
Part I:
Introduction to Queueing games

Queueing Games or Rational Queueing



Classical Queueing vs. Rational Queueing

- Classical Queueing Theory:
  * Customers: Passive entities.
  * Economic considerations: Absent.
  * Operational parameters: Given.
  * Objective: Performance evaluation of the system.

- Rational Queueing Theory = Queueing Games:
  * Customers: Active decision makers.
  * Economic considerations: Utility maximization.
  * Operational parameters: Influenced by customers' behavior.
  * Objective: Equilibrium behavior of the customers, optimization.

Classical Games vs. Queueing Games

- Classical Games:
  - Number of players: finite
  - Time: not relevant. The players are always present even if they make their decisions sequentially.
  - Utility functions: Not influenced by the congestion (number of 'active' players).

- Queueing Games:
  - Number of players (customers): infinite.
  - Time: very relevant. Players-customers arrive, make decisions and depart.
  - Utility functions: Influenced significantly by the congestion (number of present customers).

Main assumptions for tractability

- Main problems - differences from classical games:
  - * infinite potential players-customers,
  - * arrivals in an infinite time interval.

$$\downarrow$$

- Solution:
  - * Each customer plays a game against the population of customers,
  - * The utility of a tagged customer depends on
    - the strategy s of the tagged customer and
    - the steady-state behavior of the system given a fixed strategy s' of the other customers.

Queueing Games: Viewpoint and objectives

- Queueing systems are economic systems:
  - Agents (customers, servers, administrators) make decisions to maximize their utilities.

- Main objectives:
  - Describe customers' **natural** behavior (find equilibrium strategies).
  - Describe customers' **desirable** behavior (find socially optimal and profit maximizing strategies).
  - To align customers' **natural** and **desirable** behavior (invent regulation mechanisms).

Tools for optimization - regulation

- Optimization - regulation mechanisms:

  A. General economic tools:

  * Pricing (prices, taxes, subsidies),

  B. Specific queueing tools:

  * Priorities (assignment, selling, bidding),
  * **Information provision.**

Main question

- What type of information should be given to the customers of a service system to improve a certain objective (= measure of economic performance)?

A typical study

- Information structure → Customers' strategy space.
- For any population strategy s' → System performance.
- For a tagged customer's strategy s, population strategy s' and provided information i → Utility $U(s, s'|i)$.
- $\max_s U(s, s'|i)$ → Best response $BR(s'|i)$.
- Find $s \in BR(s|i)$ for all i → Equilibrium strategies s^e.
- Maximize measures under equilibrium strategies (social welfare, profit, throughput).
- Compare information structures.

The basic unobservable model

- **Edelson, N.M. and Hildebrand, K. (1975)** Congestion tolls for Poisson queueing processes. *Econometrica.*

  Join-or-balk for unobservable M/M/1 queue.
  - λ, μ: arrival, service rates. $\rho = \frac{\lambda}{\mu}$.
  - R: service value, C: waiting cost per time unit.
  - Upon arrival, a customer decides whether to join or balk knowing only the type of the queue and the various parameters.

  Application of the steps of a typical study:
  - Strategy: Join probability q.
  - Population strategy $= q' \rightarrow$ M($\lambda q'$)/M(μ)/1 queue.
  - Utility $U(q, q') = q \cdot (R - \frac{C}{\mu - \lambda q'}) + (1 - q) \cdot 0$
  - ....

The basic unobservable model - Results

- Equilibrium strategy: q_e-joining probability

Case	Equil. prob. q_e
$R \leq \frac{C}{\mu}$	0
$\frac{C}{\mu} < R < \frac{C}{\mu-\lambda}$	$\frac{\mu - C/R}{\lambda}$
$R \geq \frac{C}{\mu-\lambda}$	1

- Socially optimal strategy: q_{soc}-joining probability

Case	Soc. opt. prob. q_{soc}
$R \leq \frac{C}{\mu}$	0
$\frac{C}{\mu} < R < \frac{C\mu}{(\mu-\lambda)^2}$	$\frac{\mu - \sqrt{\mu C/R}}{\lambda}$
$R \geq \frac{C\mu}{(\mu-\lambda)^2}$	1

- $q_{prof} = q_{soc} \leq q_e$.

Information possibilities

- Classical types of information for join-or-balk decisions:
  * Number of present customers,
  * Server's state,
- Recent types of information for join-or-balk decisions:
  * Age of current service time,
  * **Decisions of previous customers.**

Information on the number of present customers

- Models for the M/M/1 queue:
  - * **Perfect** observation structure:
    The customers observe exactly the queue length.
  - * **Imperfect** observation structure:
    The customers observe imperfectly the queue length.
  - * **Delayed** observation structure:
    The customers observe the queue length with delay.
  - * **Mixed** observation structure:
    Some customers observe the queue length.
  - * **Alternating** observation structure:
    The system has observable and unobservable periods.

The basic observable model

- **Naor, P. (1969)** The regulation of queue size by levying tolls. *Econometrica.*

  Observable M/M/1 queue.

  * λ, μ: arrival, service rates. $\rho = \frac{\lambda}{\mu}$.
  * R: service value, C: waiting cost per time unit.
  * Upon arrival, a customer inspects the queue length and decides whether to join or balk.

The basic observable model - Results

- Individually optimal strategy: n_e-threshold strategy

$$n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \text{ (Naor's threshold).}$$

- Socially optimal strategy: n_{soc}-threshold strategy

$$n_{soc} = \lfloor x_{soc} \rfloor, \text{ where } \frac{x_{soc}(1-\rho) - \rho(1-\rho^{x_{soc}})}{(1-\rho^2)} = \frac{\mu R}{C}.$$

- Profit maximizing strategy: n_{prof}-threshold strategy

$$n_{prof} = \lfloor x_{prof} \rfloor, \text{ where } x_{prof} \text{ the root of another eq.}$$

- Naor's inequality: $n_{prof} \leq n_{soc} \leq n_e$.

Part II:
Informing customers about previous customers'decisions

joint work with
Odysseas Kanavetas, Athanasia Manou and Sotiris Tzinakis

Motivation

- **A new kind of information:**
  * Previous customers decisions.

- **Advantages:**
  * Smoothing of the arrival process through coordination of customers.
  * Valuable sort of information when the service system of interest is the entrance to a more general service system whose state is not known.

- **Example:**
  * A web-platform that suggests a certain e-shop and routes customers there if they follow the suggestion: It does not know the congestion in the e-shop but knows the customers who have been routed there.

I: The detailed Bernoulli information scheme

- **Info:** The customers are informed about the join-or-balk decisions of the last N more recent arrivals.
- **Example:** For $N = 3$, the information $I(t) = (0, 0, 1)$ means that the last two arrivals balked, whereas the arrival just before them entered.
- **Strategies:** A customer's strategy is a vector

$$\mathbf{q} = (q_{i_1,i_2,\ldots,i_N} : (i_1, i_2, \ldots, i_N) \in \{0, 1\}^N),$$

where $q_{i_1,i_2,\ldots,i_N}$ is the joining probability for a customer who receives the information $I(t) = (i_1, i_2, \ldots, i_N)$ regarding the decisions of the last N arrivals.

I: The detailed Bernoulli information scheme

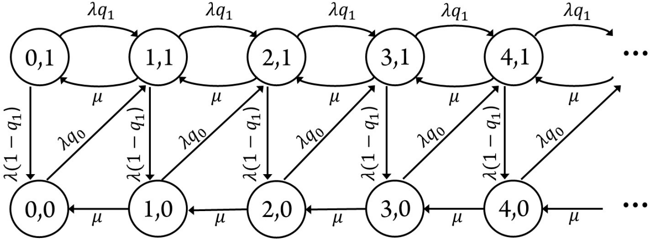
- **Stochastic Process:** Under a given strategy $\mathbf{q}$ of the population, the process $\{(Q(t), I(t))\}$ (number of cust, info) is a QBD with transition rate matrix

$$\mathbb{Q} = \begin{pmatrix} B_{0,0} & B_{0,1} & 0 & 0 & 0 & \cdots \\ B_{1,0} & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Each block is of size $2^N \times 2^N$, where N is the number of previous customers for whom the join-or-balk information is provided.

I: The detailed Bernoulli information scheme

- **Transition diagram for $N = 1$:**



- **QBD blocks for $N = 1$:**

$$A_0 = \begin{pmatrix} 0 & \lambda q_0 \\ 0 & \lambda q_1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -(\lambda q_0 + \mu) & 0 \\ \lambda(1 - q_1) & -(\lambda + \mu) \end{pmatrix},$$

$$A_2 = \mu I, \quad B_{0,0} = A_0 + A_1, \quad B_{0,1} = A_0, \quad B_{1,0} = A_2.$$

I: The detailed Bernoulli information scheme

- **QBD blocks for $N = 1$ under $\mathbf{q} = (q_0, q_1)$:**

$$A_0 = \begin{pmatrix} 0 & \lambda q_0 \\ 0 & \lambda q_1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -(\lambda q_0 + \mu) & 0 \\ \lambda(1 - q_1) & -(\lambda + \mu) \end{pmatrix},$$
$$A_2 = \mu I, \quad B_{0,0} = A_0 + A_1, \quad B_{0,1} = A_0, \quad B_{1,0} = A_2.$$

- **QBD blocks for $N = 2$ under $\mathbf{q} = (q_{00}, q_{01}, q_{10}, q_{11})$:**

$$A_0 = \left(\begin{array}{cc|cc} 0 & 0 & \lambda q_{00} & 0 \\ 0 & 0 & \lambda q_{01} & 0 \\ \hline 0 & 0 & 0 & \lambda q_{10} \\ 0 & 0 & 0 & \lambda q_{11} \end{array}\right),$$
$$A_1 = \left(\begin{array}{cc|cc} -(\lambda q_{00} + \mu) & 0 & 0 & 0 \\ \lambda(1 - q_{01}) & -(\lambda + \mu) & 0 & 0 \\ \hline 0 & \lambda(1 - q_{10}) & -(\lambda + \mu) & 0 \\ 0 & \lambda(1 - q_{11}) & 0 & -(\lambda + \mu) \end{array}\right),$$
$$A_2 = \mu I, \quad B_{0,0} = A_0 + A_1, \quad B_{0,1} = A_0, \quad B_{1,0} = A_2.$$

II: The geometric "run-of-1s" information scheme

- **Info:** The customers are informed about the number of joining customers since the last balking customer.
- **Example:** The information $I(t) = 5$ means that the last 5 arrivals entered whereas the arrival just before them did not enter.
- **Strategies:** A customer's strategy is a sequence

$$\mathbf{q} = (q_0, q_1, q_2, \ldots),$$

where q_i is the joining probability for a customer who receives the information $I(t) = i$.

II: The geometric "run-of-1s" information scheme

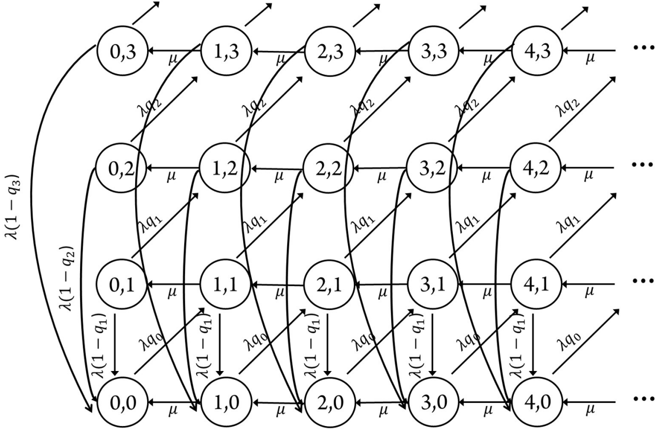
- **Stochastic Process:** Under a given strategy $\mathbf{q}$ of the population, the process $\{(Q(t), I(t))\}$ (number of cust, info) is a QBD with transition rate matrix

$$\mathbb{Q} = \begin{pmatrix} B_{0,0} & B_{0,1} & 0 & 0 & 0 & \cdots \\ B_{1,0} & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Each block is of infinite size since $I(t)$ takes values $0, 1, 2, \ldots$.

II: The geometric "run-of-1s" information scheme

- **Transition diagram:**



II: The geometric "run-of-1s" information scheme

- **QBD blocks under $\mathbf{q} = (q_0, q_1, q_2, \ldots)$:**

$$A_0 = \begin{pmatrix} 0 & \lambda q_0 & 0 & 0 & \cdots \\ 0 & 0 & \lambda q_1 & 0 & \cdots \\ 0 & 0 & 0 & \lambda q_2 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -(\lambda q_0 + \mu) & 0 & 0 & 0 & \cdots \\ \lambda(1-q_1) & -(\lambda+\mu) & 0 & 0 & \cdots \\ \lambda(1-q_2) & 0 & -(\lambda+\mu) & 0 & \cdots \\ \lambda(1-q_3) & 0 & 0 & -(\lambda+\mu) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$A_2 = \mu I, \quad B_{0,0} = A_0 + A_1, \quad B_{0,1} = A_0, \quad B_{1,0} = A_2.$$

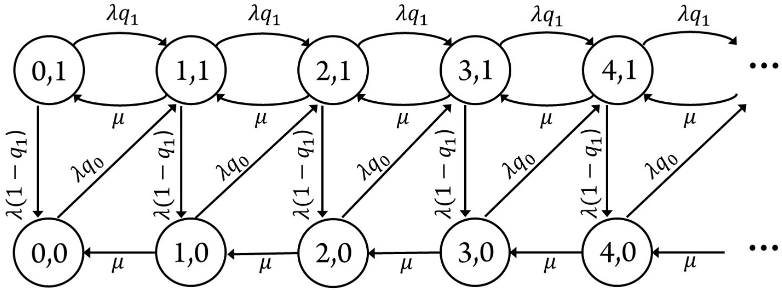
Other information schemes

- **The geometric "run-of-0s" information scheme:** The customers are informed about the number of balking customers since the last joining customer.
- **Truncated geometric "run-of-1s" or "run-of-0s" information schemes:** The customers are informed about the number of balking customers since the last joining customer being $0, 1, \ldots, N$ or 'above N'.
- **Binomial information scheme:** The customers are informed about the number of joining customers among the last N more recent arrivals.

Solving for the case $N = 1$ of the detailed Bernoulli information scheme

The Markovian model

- $Q(t)$: Number of customers at time t.
- $I(t)$: Decision of the last customer before time t.
- Under a strategy (q_0, q_1), $\{(Q(t), I(t))\}$ is a continuous time Markov chain with transition diagram



Net benefit functions

- $U(q_0, q_1|i)$: Utility of a tagged customer who sees the decision $I = i$ for the last customer and decides to join, given that the other customers follow (q_0, q_1).
- Formulas:

$$U(q_0, q_1|0) = R - \frac{C}{\mu}\left(\frac{1}{1-\rho_2} - \frac{\rho\rho_2 q_1}{\rho^2 q_0 + \rho(1-\rho_2)q_1}\right),$$

$$U(q_0, q_1|1) = R - \frac{C}{\mu}\left(\frac{1}{1-\rho_2} + \frac{\rho(\rho-\rho_2)q_0 + \rho q_1 - \rho_2}{\rho(\rho-\rho_2)q_0 + \rho q_1}\right),$$

with $$\rho_2 = \frac{\lambda\mu q_0 + \lambda\mu + \mu^2 - \sqrt{\Delta}}{2\mu^2},$$

$$\Delta = (\lambda\mu q_0 - \lambda\mu + \mu^2)^2 + 4\lambda\mu^3(1-q_1).$$

- Basic fact: $U(q_0, q_1|1) < U(q_0, q_1|0)$ for any $(q_0, q_1) \in [0,1]^2$.

Equilibrium strategies

- A unique equilibrium exists. Let $\rho = \frac{\lambda}{\mu} < 1$, $\nu = \frac{R\mu}{C}$.
- Critical values ν_1, ν_2, ν_3 exist with $0 < 1 < \nu_1 < \nu_2 < \nu_3$:

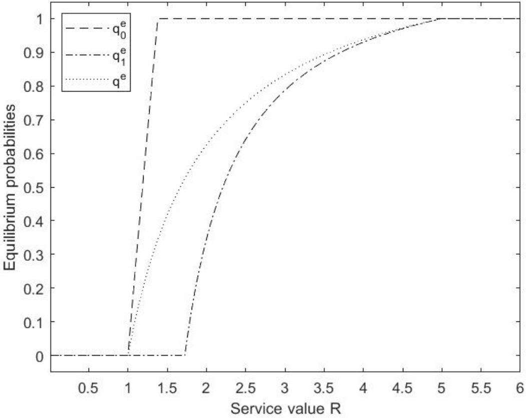
Value of $\nu \in$	$[0,1]$	$(1,\nu_1)$	$[\nu_1,\nu_2]$	(ν_2,ν_3)	$[\nu_3,\infty)$
Equil. strat.	$(0,0)$	$(q_0^*,0)$	$(1,0)$	$(1,q_1^*)$	$(1,1)$

- ν_1, ν_2, ν_3 and q_0^*, q_1^* are given as:

$$\begin{aligned}
\nu_1 &= \frac{2}{1-2\rho+\sqrt{1+4\rho}}, \\
\nu_2 &= \frac{5\rho+1-(\rho+1)\sqrt{1+4\rho}}{3\rho-\rho\sqrt{1+4\rho}}, \\
\nu_3 &= \frac{1}{1-\rho}, \\
q_0^* &= \text{unique solution of } U(x,0|0)=0, \\
q_1^* &= \text{unique solution of } U(1,x|1)=0.
\end{aligned}$$

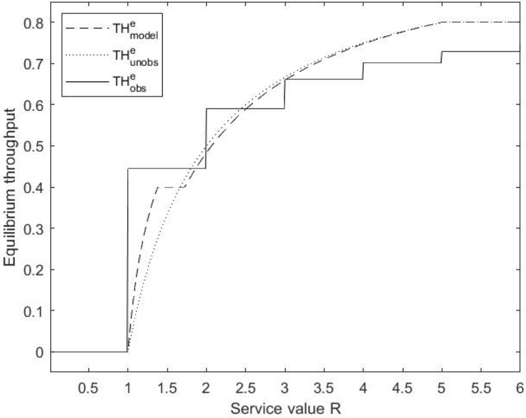
Comparison of equilibrium strategies

- Numerical scenario: $\lambda = 0.8$, $\mu = 1$, $R \in [0, 6]$, $C = 1$.



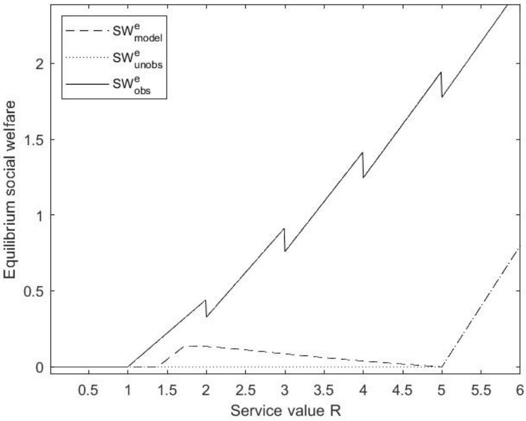
Comparison of equilibrium throughputs

- Numerical scenario: $\lambda = 0.8$, $\mu = 1$, $R \in [0, 6]$, $C = 1$.



Comparison of equilibrium social welfare functions

- Numerical scenario: $\lambda = 0.8$, $\mu = 1$, $R \in [0, 6]$, $C = 1$.



Some qualitative findings

- The equilibrium joining probability for the unobservable model is always between the joining probabilities of the present model.
- The present model resides between the unobservable and the observable model regarding the equilibrium throughput and the equilibrium social welfare.
- The social welfare improvement over the unobservable model is maximized for the range of the parameters where the equilibrium strategy is $(1, 0)$.

References I

- General references for queueing games:

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  * Economou, A. (2024) The impact of information about last customer's decision on the join-or-balk dilemma in a queueing systems. *Annals of Operations Research.*

  * Economou, A. (2021) The impact of information on strategic behavior in queueing systems. Chapter 4 in *Anisimov, V. and Limnios, N. (2021) Queueing Theory 2, Advanced Trends.* Wiley/ISTE.

Thank you!

Questions?