Optimal outpatient appointment scheduling with emergency arrivals

Paulien Out

January 14, 2010

Joint work with Ger Koole

The problem

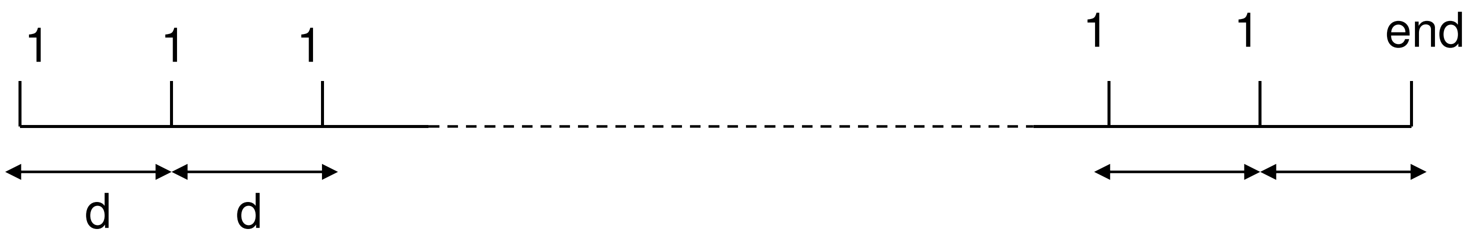
Scheduling appointments

- Situation:
  - block of time available to schedule N patients
  - stochastic service times (known distribution)
  - unknown number of emergency arrivals

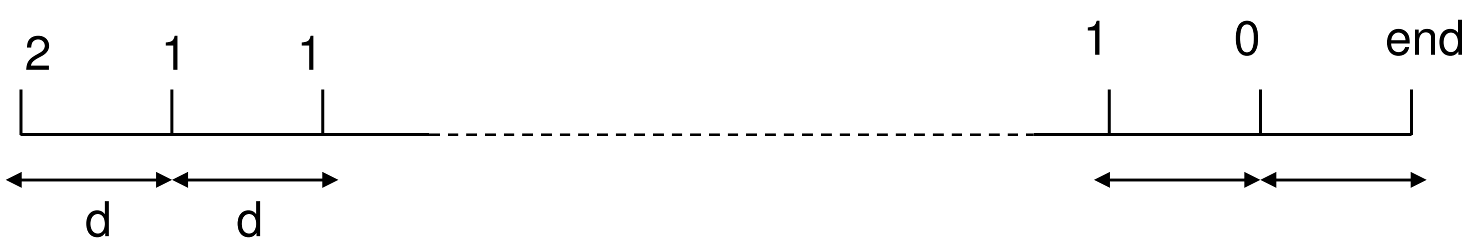
- At what times to schedule appointments?
- What waiting time, idle time, tardiness?

Standard practice

Schedule patients at even-spaced intervals:



Sometimes Bailey-Welch rule:



Earlier work

Guido Kaandorp and Ger Koole (2007):

- exponential service times
- no emergencies
- local search method
- optimal schedule based on weighted sum of waiting times, idle time and tardiness

Earlier work (2)

Vanden Bosch, Dietz and Simeoni (1999)

- Erlang service times
- no emergencies
- efficient algorithm: number of evaluations linear in number of intervals and patients
- but less efficient then it appears due to a small error in the proof

The model

Variables:

– T intervals of length d

– N patients to be scheduled

– mean service time β

– decision variables:

x_t: number of patients scheduled at interval t

t = 1, 2, …, T

Adding emergency patients

- Unknown number of arrivals
  - mean number λ per day is known
- Arrivals according to Poisson process
- Possibly different service time distribution
- In model:
  - arrive only at start of intervals
  - number per interval has Poisson distribution

Service of emergencies

- All patients wait for patient in service
- Scheduled patients wait for work present upon their arrival and any emergency arriving during their waiting time
- Emergency patients wait only for each other

Objective

Schedule all N patients while minimising a weighted combination of:

- waiting time per scheduled patient
- idle time
- tardiness

States

- State: number of minutes of work present
- For times 0, 1, ..., T

$p_t^-(i)$ =P(state=i just before time t)

$p_t^+(i)$ =P(state=i just after time t)

State transitions

$$p_0^-(0) = 1$$

$$p_t^-(x-d) = p_{t-1}^+(x) \qquad x > d$$

$$p_t^-(0) = \sum_{i=0}^{d} p_{t-1}^+(i)$$

$$p_t^+(x) = \sum_{i=0}^{x} p_{t-1}^-(i) P\left(arriving \quad work \quad = \quad x-i\right)$$

Tardiness

$$L(x) = \sum_{k=0}^{\infty} k p_T^-(k)$$

expected amount of work present at end of day

Server idle time

$$I(x) = Td + L(x) - N\beta_1 - \lambda\beta_2$$

Expected makespan minus expected total service time

Waiting time

- Waiting time consists of three parts:
- Work present upon arrival
- Patients arriving simultaneously but served earlier
- Emergencies arriving during waiting time
- Loop over following intervals and compute convolutions of waiting time and amount of emergency work arriving

Solution method: local search

- Neighbourhood: all possible combinations of 1-interval shifts
- Start from any possible solution

  (for example (N,0,…,0))
- For speed: first use neighbourhood of all single 1-interval shifts

Multimodularity

- Related to convexity, for functions defined on (a subset of) Z^m
- Proof by coupling
- If a function is multimodular, local search always converges to optimal solution

Result by Koole & VD Sluis (2003)

Speed

- Seconds for small problems (up to 15 intervals) to a few hours for larger problems (36 intervals)
- Acceptable because not meant to be executed throughout the day
- Neighbourhood size is 2^T-2
- Not a PLS problem

Example: improving standard practice

- T=18, d=10
- Service times exponential with mean 30

	Schedule	Waiting time	Idle time	Tardi-ness
optimal	1-1-0-1-0-1-0-1-0-0-0-0-0-0-0-0-0-0	31.57	9.96	16.18
standard	1-0-0-1-0-0-1-0-0-0-1-0-0-0-1-0-0-0	14.61	42.23	23.68

Example: influence of emergencies

- T=18, d=10
- Service times exponential with mean 20

Number scheduled	Nr of emergen-cies	Schedule	Waiting time	Idle time	Tardi-ness
8	0	2-0-1-0-1-0-1-0-1-0-0-1-0-1-0-0-0-0	22.79	36.80	16.78
6	2	1-1-0-1-0-0-1-0-0-1-0-0-1-0-0-0-0-0	26.41	41.86	21.85

Future work

- Add late and/or early arrivals
- Extension of model to more than one server
- Adding more patient types with different urgency and/or service durations
- Case study in real-world setting

Thanks for your attention

Questions? Remarks? Suggestions?

paulien@few.vu.nl