On the Power and Limitations of Affine Policies in Dynamic Optimization

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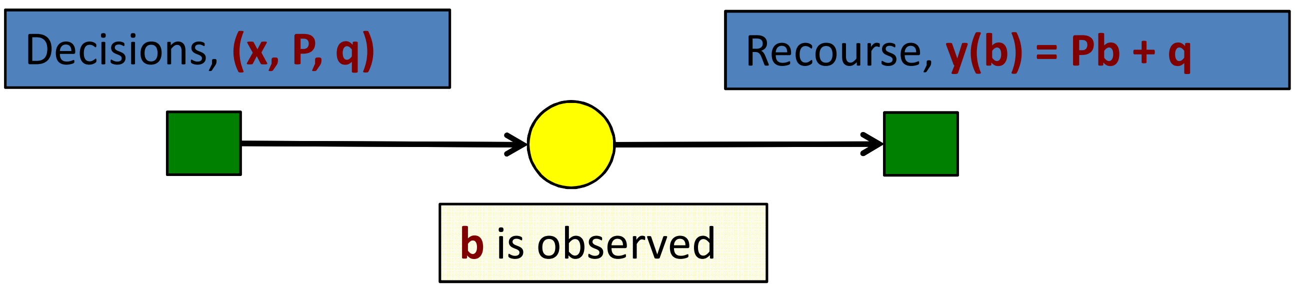
Two Stage Adaptive Optimization

$$
\begin{aligned}
\textbf{zAdapt} = \min c^T x + \max_{b \in \mathcal{U}} \; d^T y(b) & \\
Ax + By(b) & \geq b, \; \forall b \in \mathcal{U} \\
x, y(b) & \geq 0
\end{aligned}
$$

Affine Policies

$$
\begin{aligned}
\textbf{zAdapt} = \min c^T x + \max_{b \in \mathcal{U}} \; d^T y(b) & \\
Ax + By(b) &\geq b, \; \forall b \in \mathcal{U} \\
x, y(b) &\geq 0
\end{aligned}
$$

$$y(b) = Pb + q$$ → (Affine function of RHS, b)



Why Affine?

- Tractable policies: Polynomially computable

- Extend to the multi-stage case (also polynomially computable)

- There are special cases that are optimal

- Excellent practical performance

Affine Policies: Previous Work

- Extensively studied in literature
  - Gartska and Wets (1974), Rockafellar and Wets (1978)
  - Bemporad and Morari (1999)
  - Bertsimas et al. (2009), Skaf and Boyd (2009)

- Perform extremely well in practice
  - Kalman filtering (Kalman (1960))
  - Linear decision rules for approximate DP (Bertsekas (2001), de Farias and Van Roy (2003))
  - Retailer-supplier flexible commitment contracts (Ben-Tal et al. (2005))

Affine Policies: Simplex Uncertainty Sets

Simplex $$\boxed{\mathcal{U} = \textbf{conv}(b^1, \ldots, b^{m+1})}$$

Affine policies are optimal if the uncertainty set is a simplex

$$y(b) = \underbrace{\begin{bmatrix} & P & \end{bmatrix}}_{\text{m columns}} b + \begin{bmatrix} q \end{bmatrix}$$

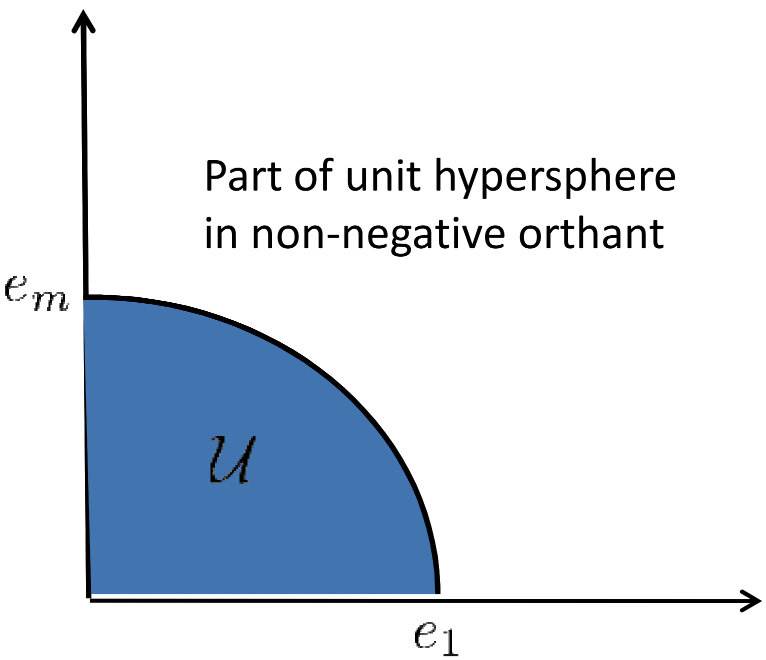
- Simplex has (m+1) extreme points
- Enough **degrees of freedom** to obtain an **optimal solution**

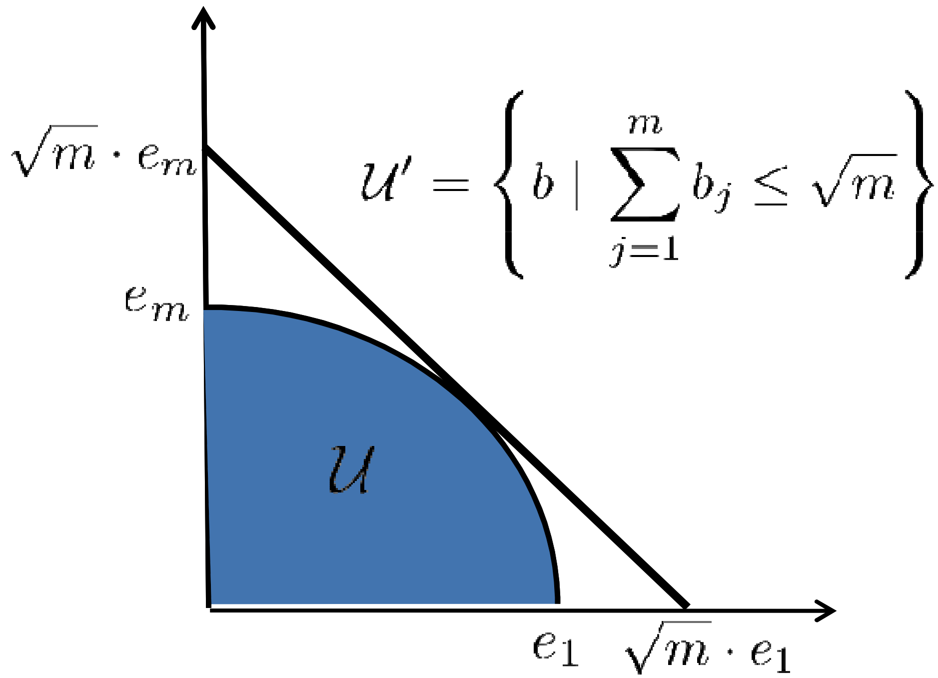
Affine Policies: General Convex Sets

- **Cost of optimal affine policy is at most $\sqrt{m}$ times the optimal adaptive problem (zAdapt)**
- **Cost of optimal affine policy is at least $\Omega(\sqrt{m})$ times the optimal adaptive problem (zAdapt)**

Performance of affine policies $\Theta(\sqrt{m})$ times the optimal

Geometric Intuition





Conclusions

- **Affine policies have both power and limitations**
- **They are tractably computed.**
- **Extensions to polynomial policies seem promising.**